Tests of QED

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How to test QED theory

- Precision tests of the **Quantum Electrodynamics** theory usually consist in the measurement of the electromagnetic fine structure constant α in different systems. Experimental results are compared with theoretical predictions
- The validation process requires **very high precision** in both measurements and theoretical calculations
- QED is then confirmed to the extent that these measurements of α from different physical sources agree with each other
- The most stringent test of QED is given by the measurement of the **electron magnetic moment**. However, several other experimental tests have been performed in different energy ranges ranges and systems:
  - Low energy range, accessible with small experiments
  - High energy range, accessible with particle colliders (e.g. e^+e^- colliders)
  - Condensed matter systems
- As we will see, the achieved precision makes QED one of the **most accurate physical theories** constructed so far

Part one:

Tests of QED at high energy

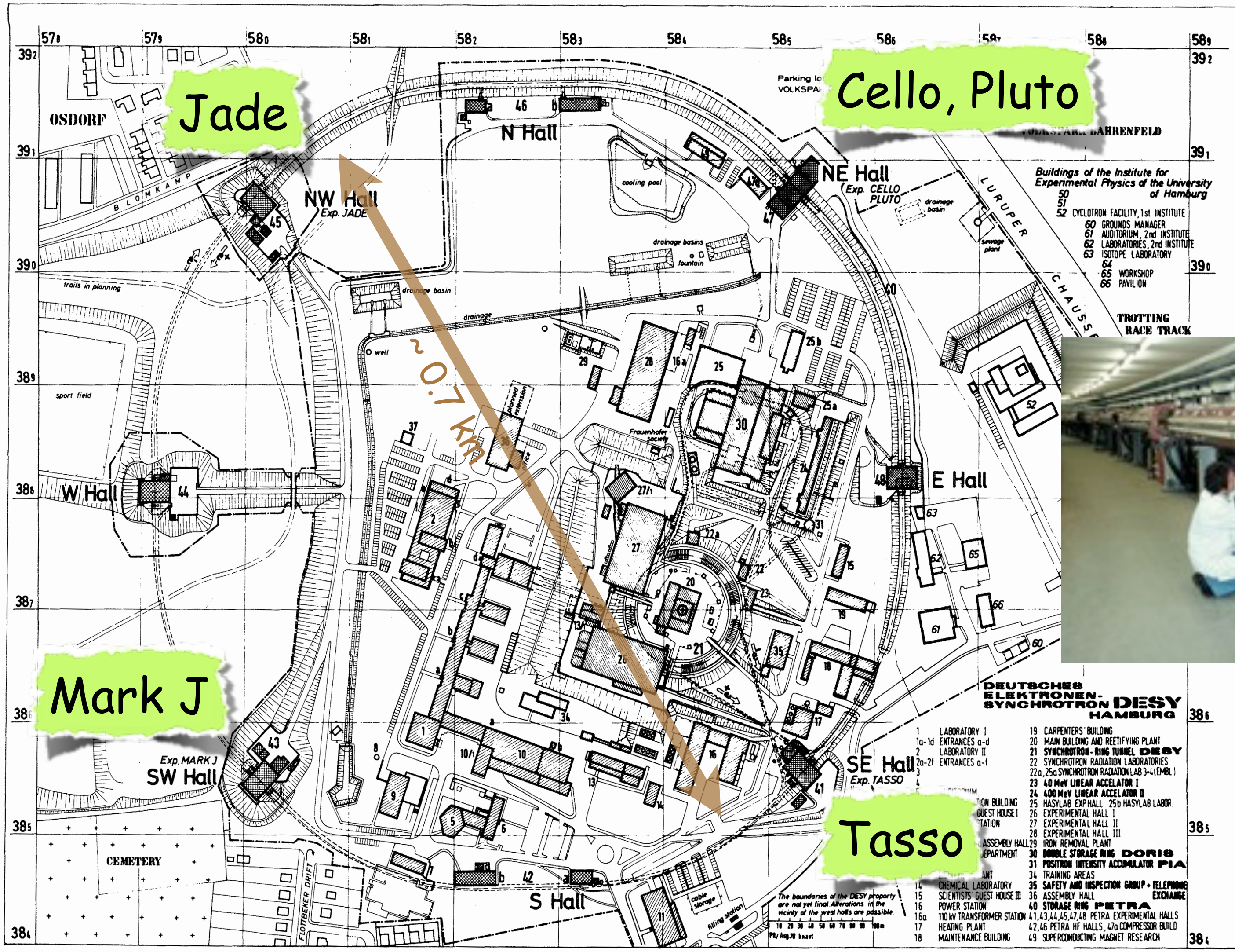
Tests of QED at high energy

- In addition to the low-energy experiments, QED has been tested also in high energy electron-positron collisions
- We discuss here the following reactions:
  - $e^+e^- \to e^+e^-$ *(Bhabha scattering)*
  - $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ *(Lepton pair production)*
  - $e^+e^- \to \gamma\,\gamma$ *(Two photon annihilation)*
  - $e^+e^- \to q\,\bar{q} \to$ hadrons *(Total hadronic cross section)*
- The energy range ($\sqrt{s}$) between 12 GeV and 47 GeV was investigated with the PETRA accelerator at DESY (Hamburg)
- High energy ranges (90-200 GeV) were covered by the LEP collider at CERN (Geneva). However, electro-weak contributions to the cross-sections become considerable at these energies
- Intermediate energies were covered by TRISTAN and SLC

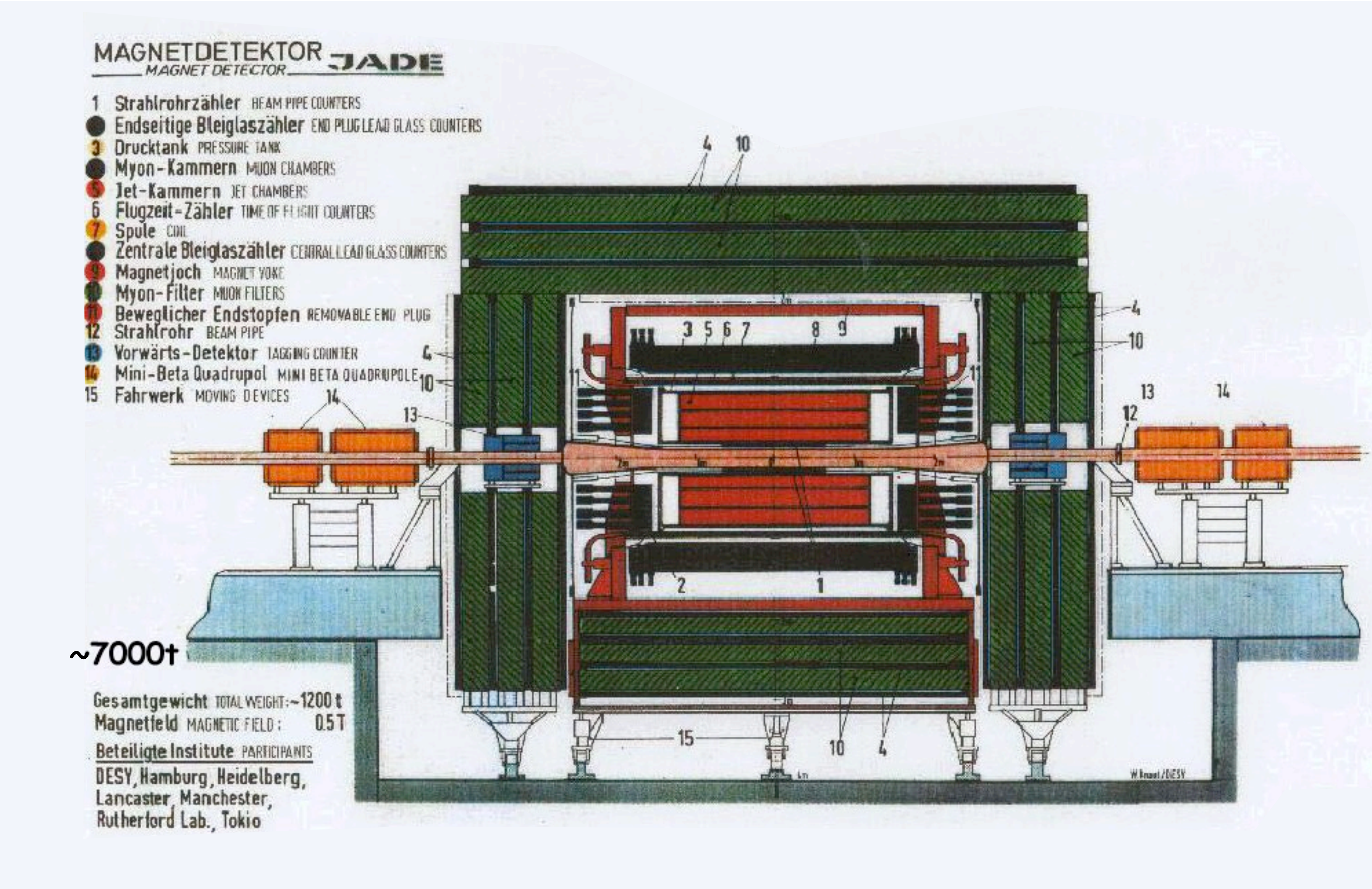
e+e- colliders

Accelerator	Experiments	CMS-Energy	Integrated luminosity
SPEAR	SPEAR	2 - 8 GeV	—
PEP	ASP, DELCO, HRS, MARK II, MAC	up to 29 GeV	~ 300 pb^{-1}
PETRA	JADE, MARK J PLUTO, TASSO CELLO	12 – 47 GeV	~ 20 pb^{-1}
TRISTAN	TRISTAN	50 – 60 GeV	~ 20 pb^{-1}
SLC	MARK II, SLD	90 GeV	~ 25 pb^{-1}
LEP	ALEPH, DELPHI, OPAL, L3	90 – 200 GeV	~ 200 pb^{-1} ~ 700 pb^{-1}

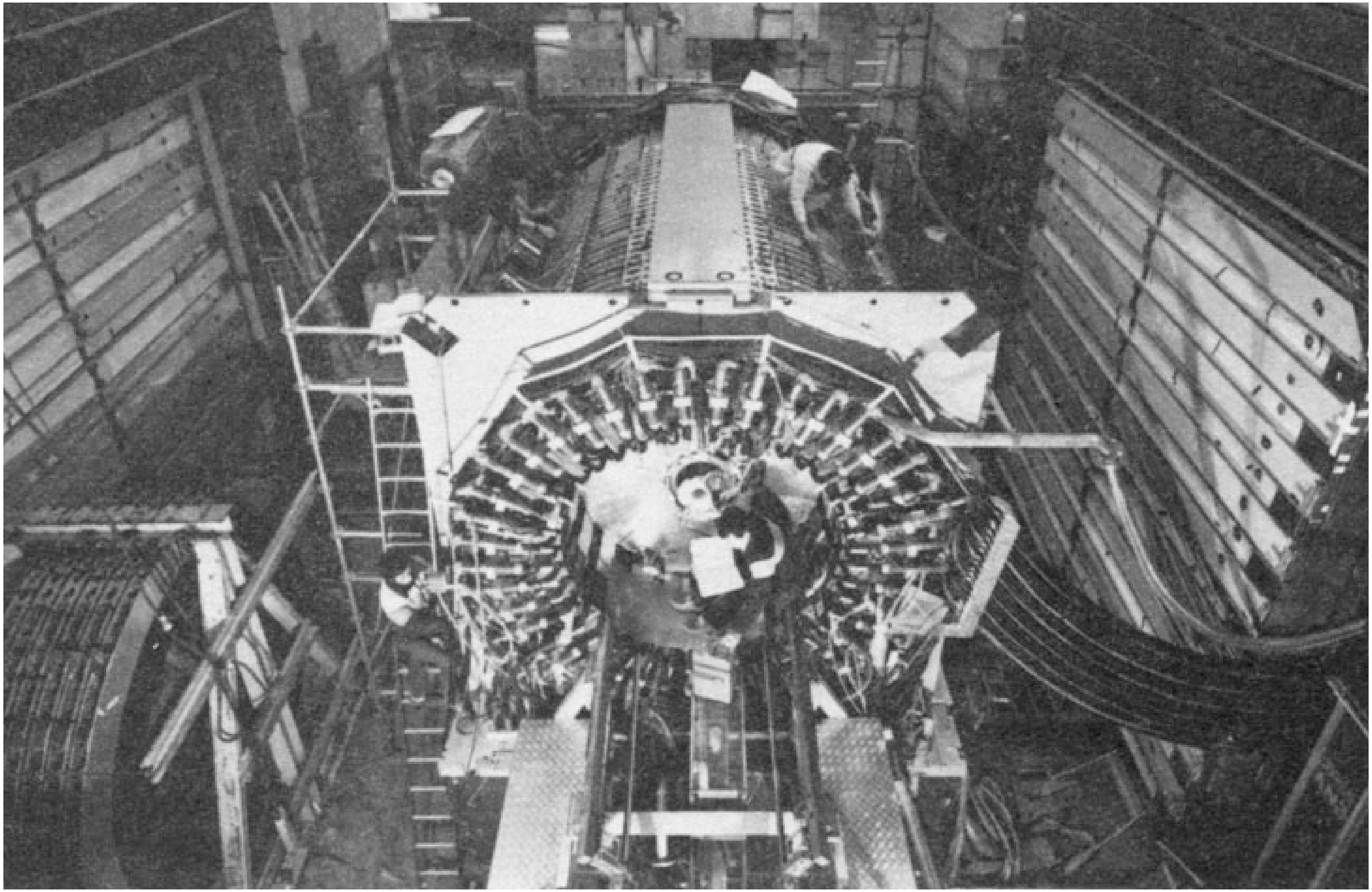
PETRA e+e- storage ring



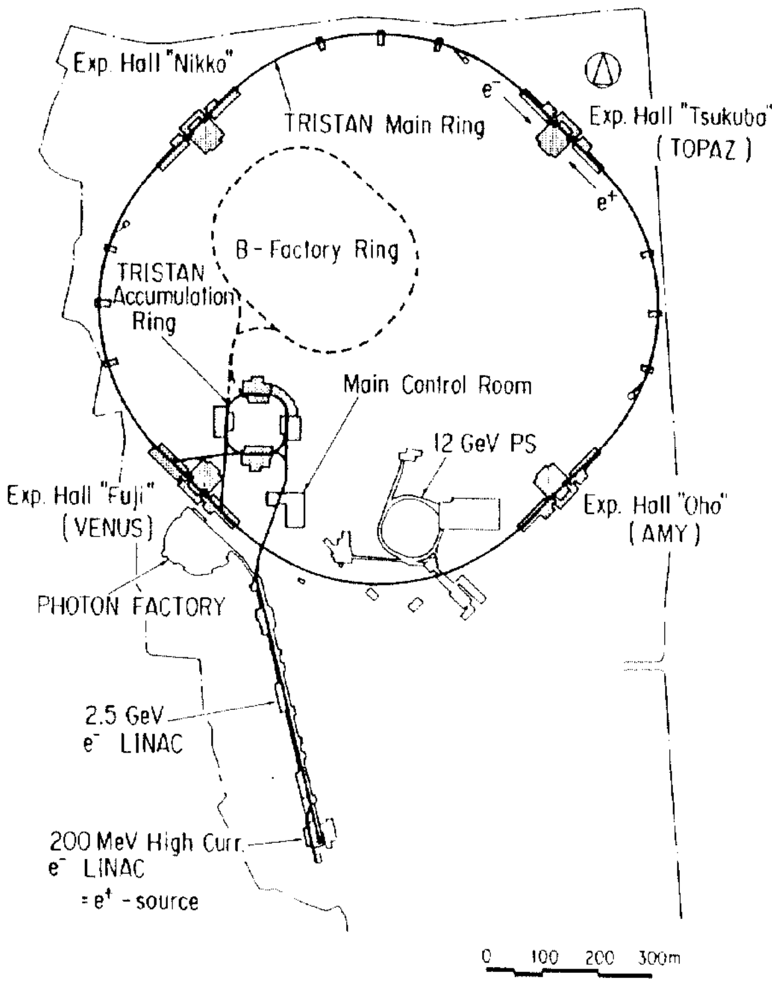
Example: JADE detector



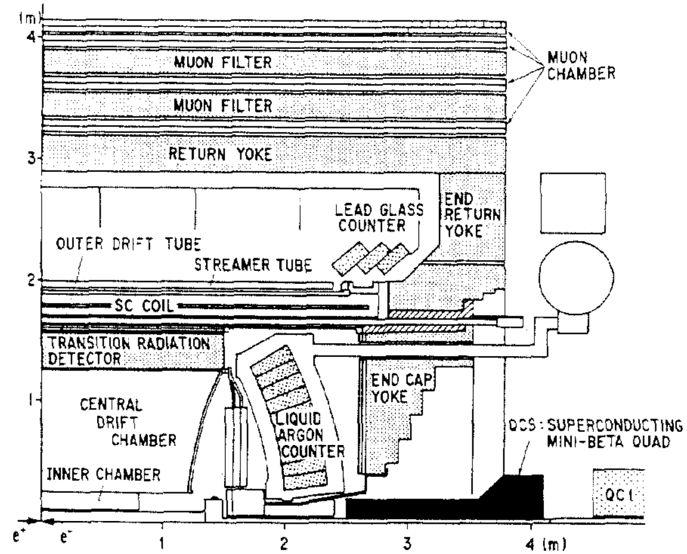
Example: JADE detector



TRISTAN accelerator & detector

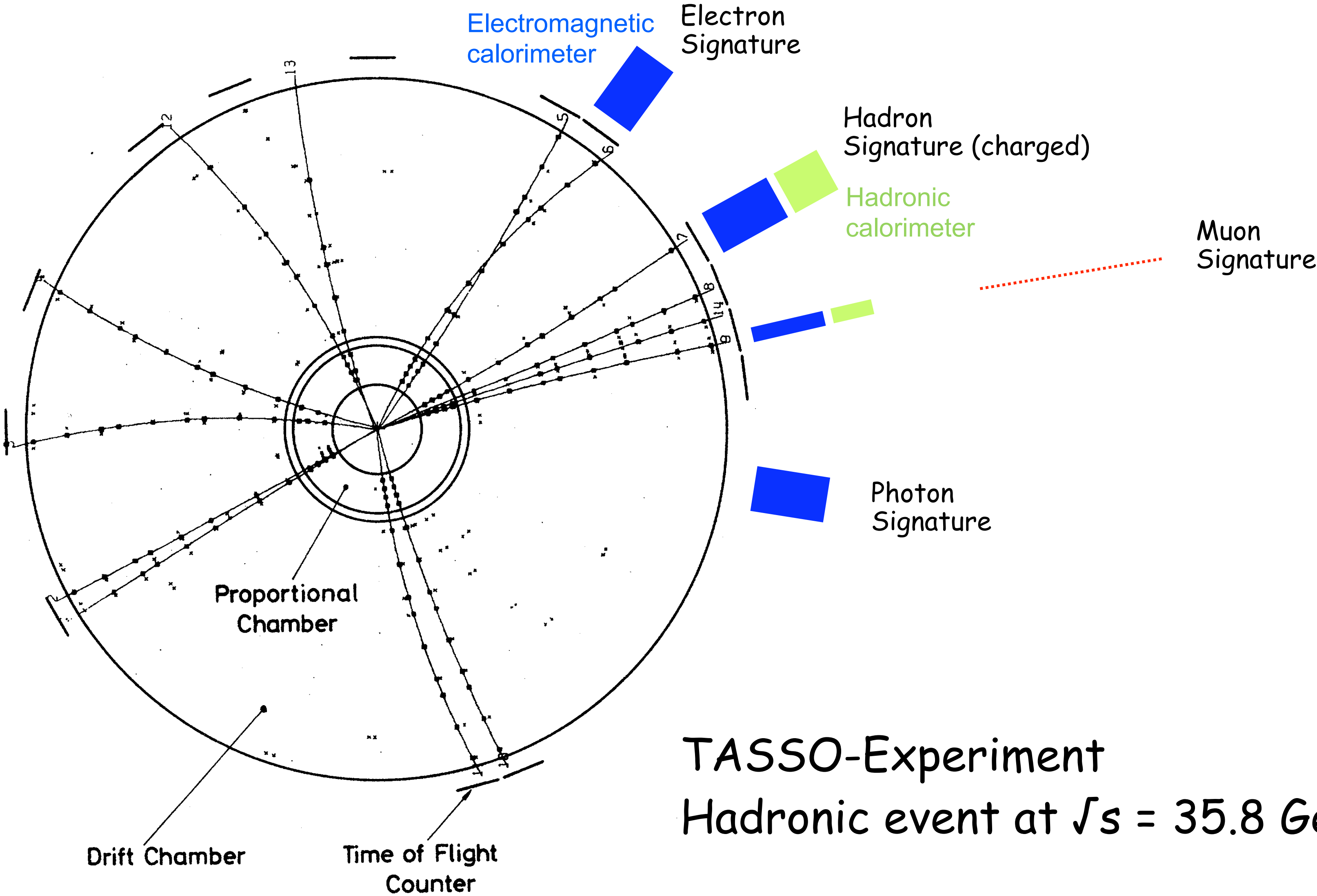


Schema of the detector



Max. beam energy: 32 GeV
Injection energy: 8 GeV
Beam lifetime: 5-6 hr.
Peak luminosity: 1.4×10^{31} $cm^{-2}s^{-1}$

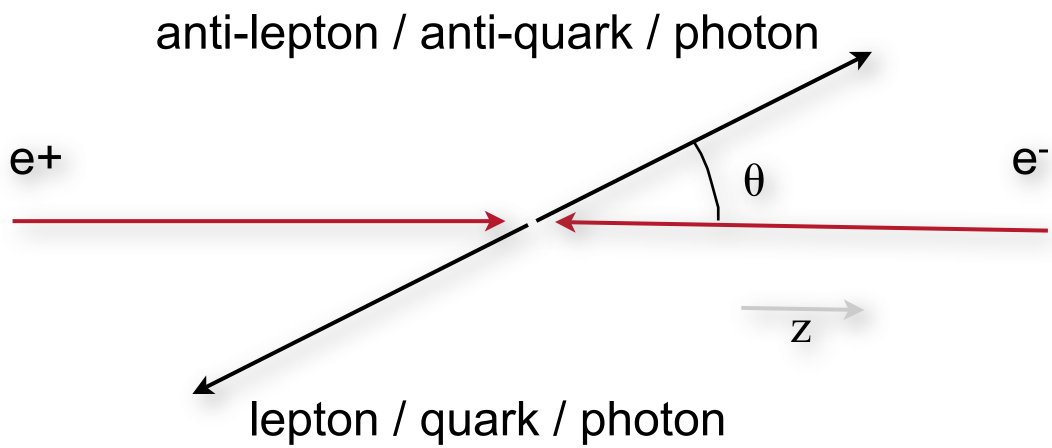
Event reconstruction



TASSO-Experiment

Hadronic event at $\sqrt{s}$ = 35.8 GeV

Reminder: e^+e^- kinematics



Kinematic variables

$$q^2 = -s\frac{(1-\cos\theta)}{2} \quad u$$

$$q'^2 = -2\frac{(1+\cos\theta)}{2} \quad t$$

$$\beta = \frac{p}{E}$$

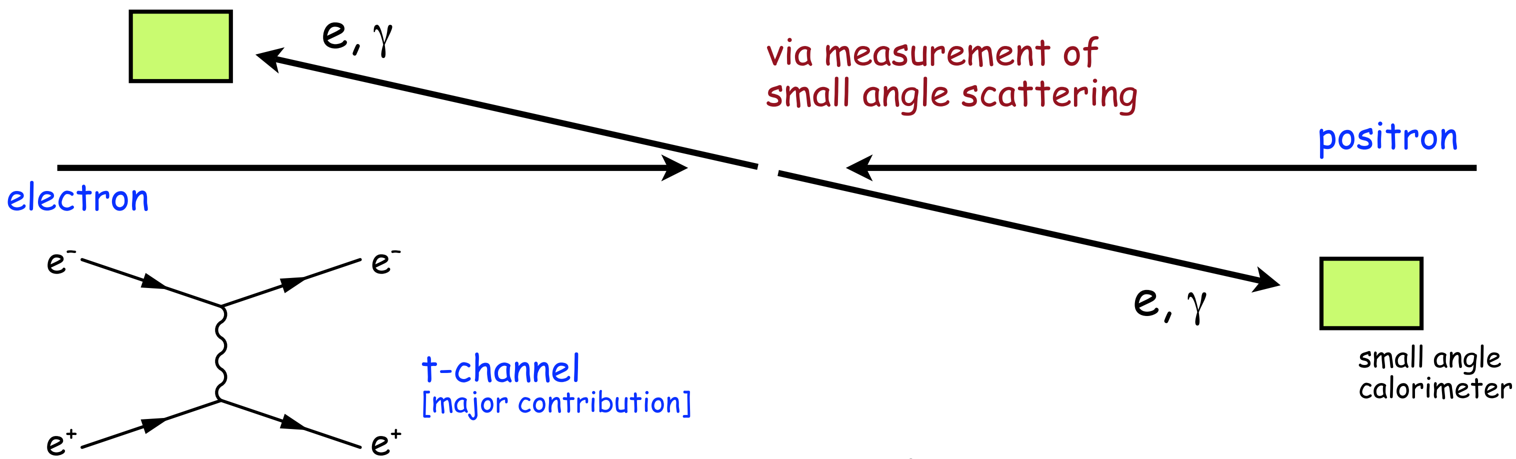
Expected cross section

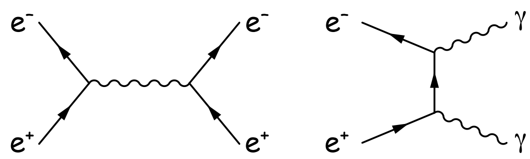
$$\frac{d\sigma_{QED}}{d\Omega} = \frac{d\sigma_0}{d\Omega}(1+\delta_{rad})$$

Radiative (higher order) corrections

How do we measure a cross section?

- To measure a cross section we divide the measured number of events by the integrated luminosity: $\sigma = N/L$



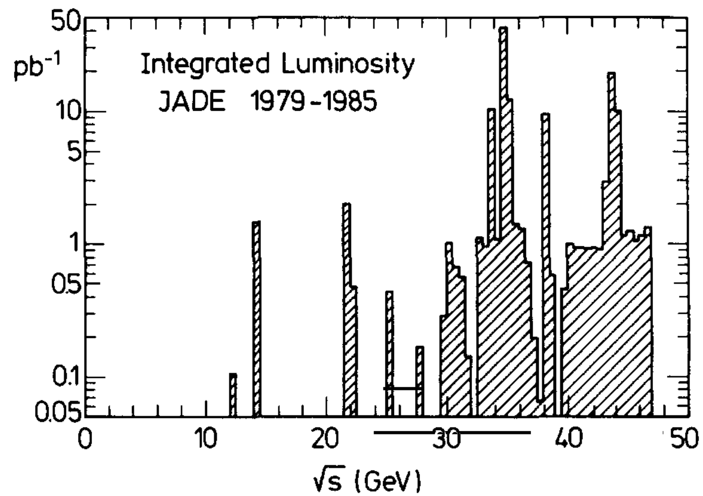


Measure

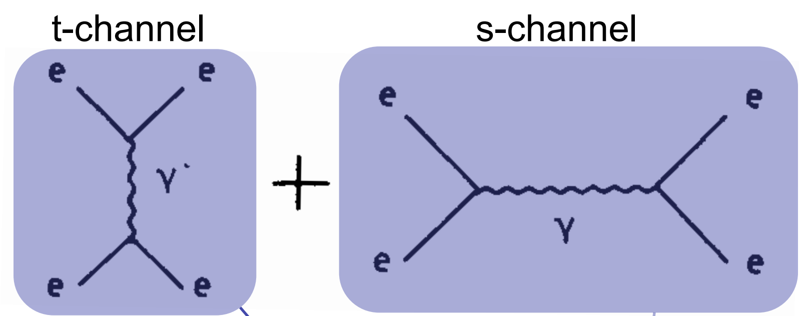
Calculate

$$\sigma^{\text{theo}}_{ee,\gamma\gamma} = \frac{N(1-b)}{(\epsilon A)\cdot L}$$

Determine



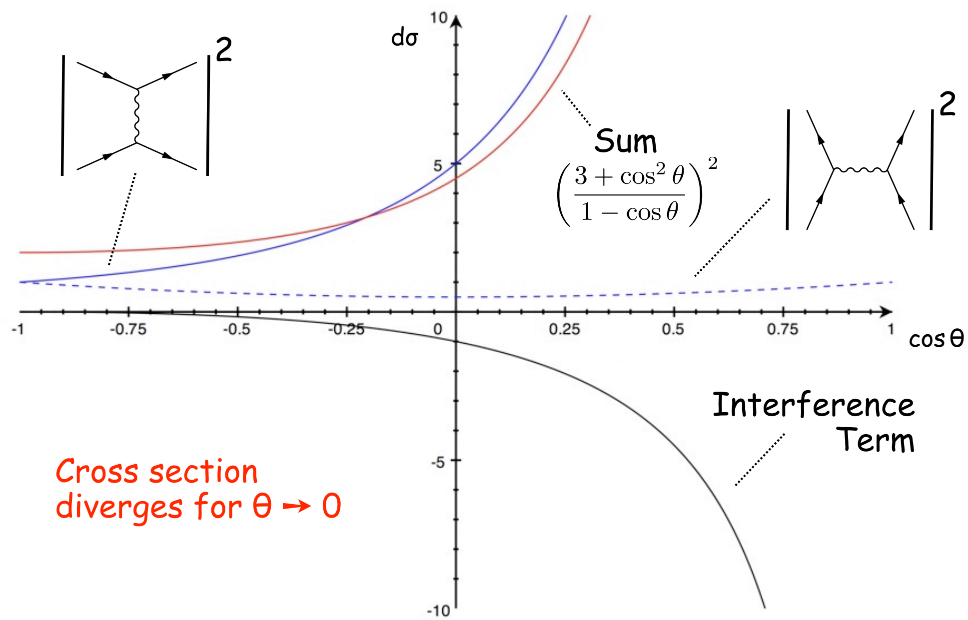
Bhabha scattering



interference term

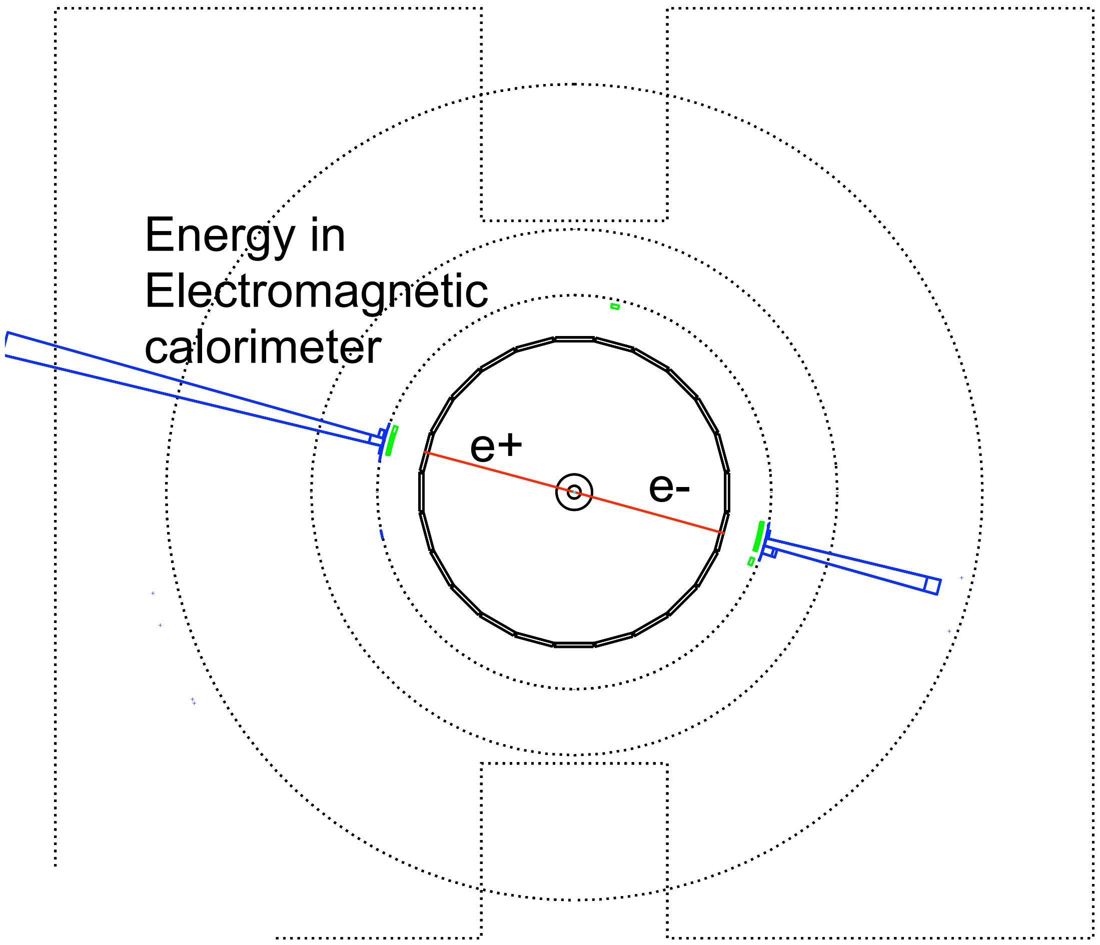
$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{q'^4 + s^2}{q^4} + \frac{2q'^4}{q^2 s} + \frac{q'^4 + q^4}{s^2}\right)$$

$$= \frac{\alpha^2}{4s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

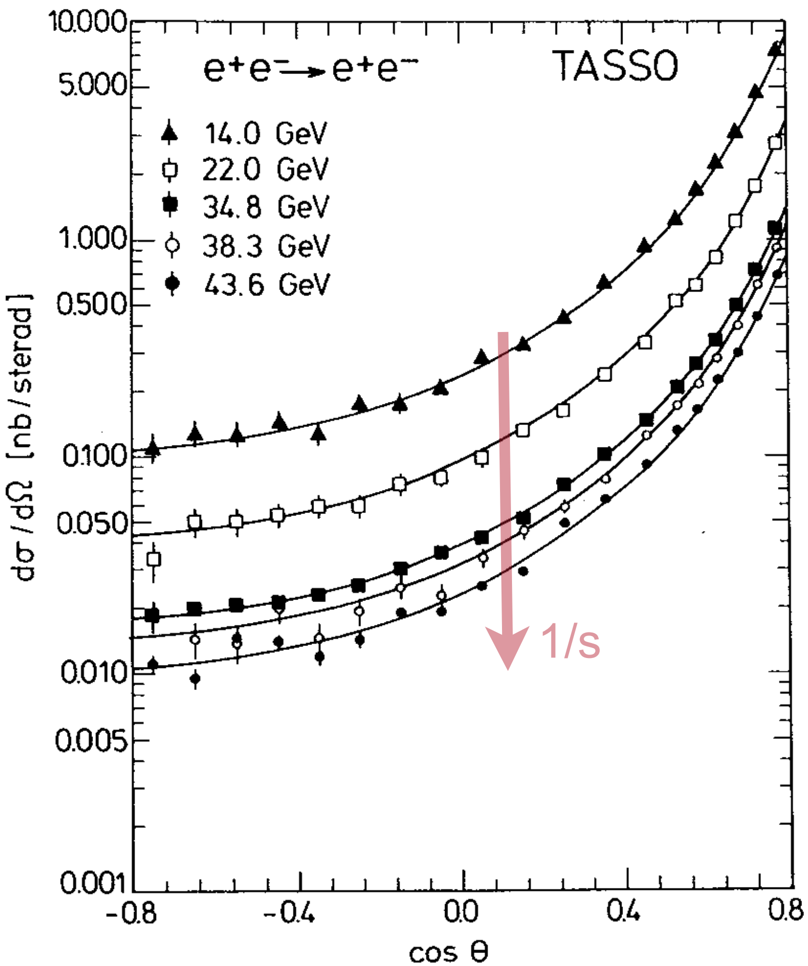


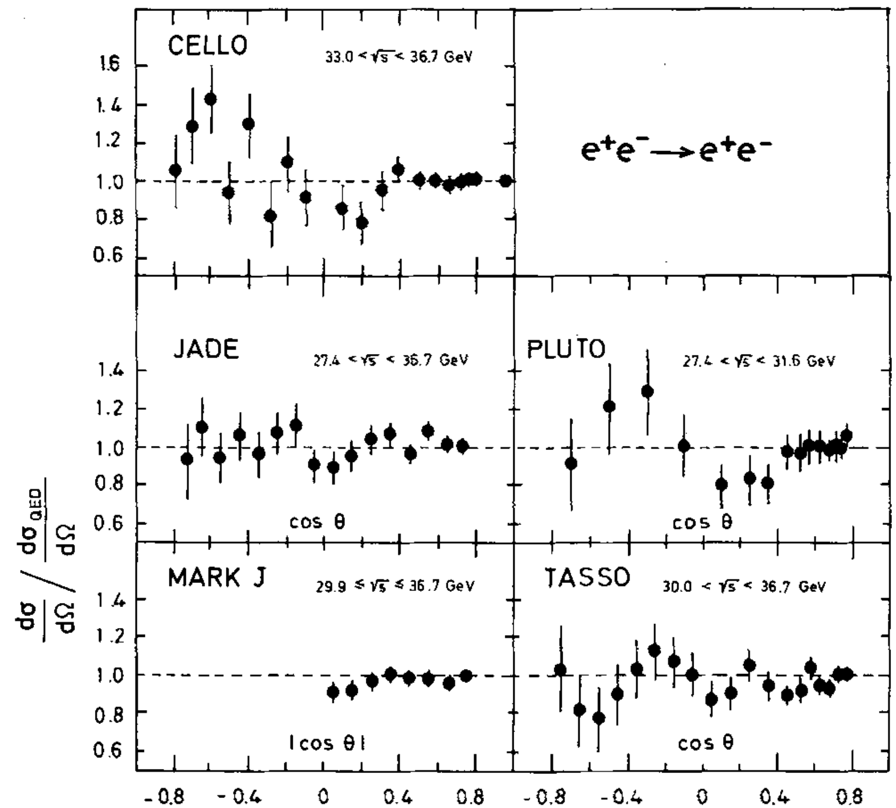
- Leading order cross section divergent for cos(θ)=1, i.e. for θ=0.

Example: event display



Bhabha scattering





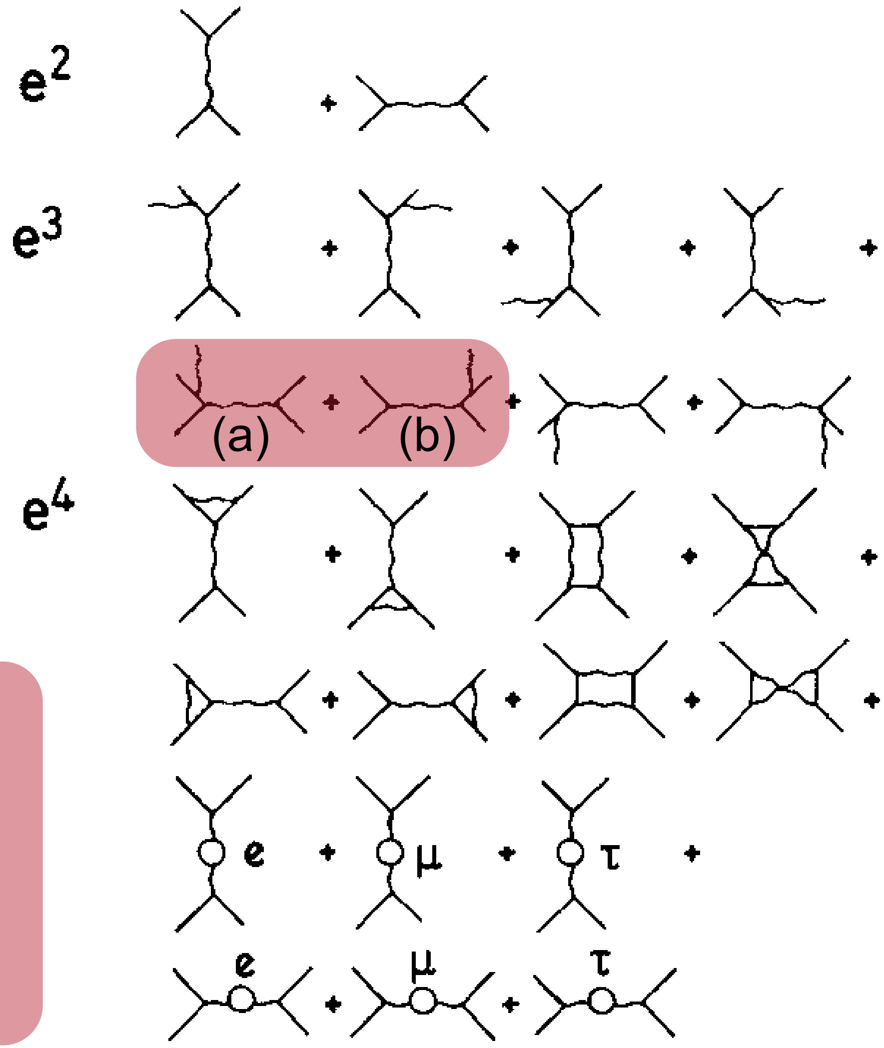
Ratio between measured cross section and QED calculation

Higher order corrections

$\propto \alpha^2 = (1/137)^2 = 5\text{x}10^{-5}$

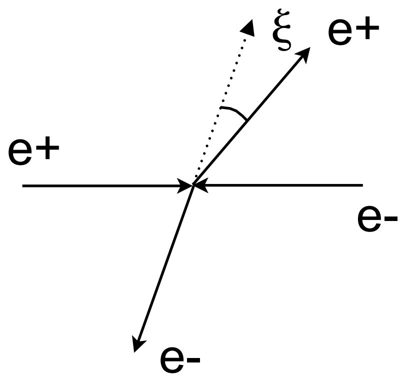
$\propto \alpha^3 = (1/137)^3 = 4\text{x}10^{-7}$

$\propto \alpha^4 = (1/137)^4 = 2.8\ 10^{-9}$

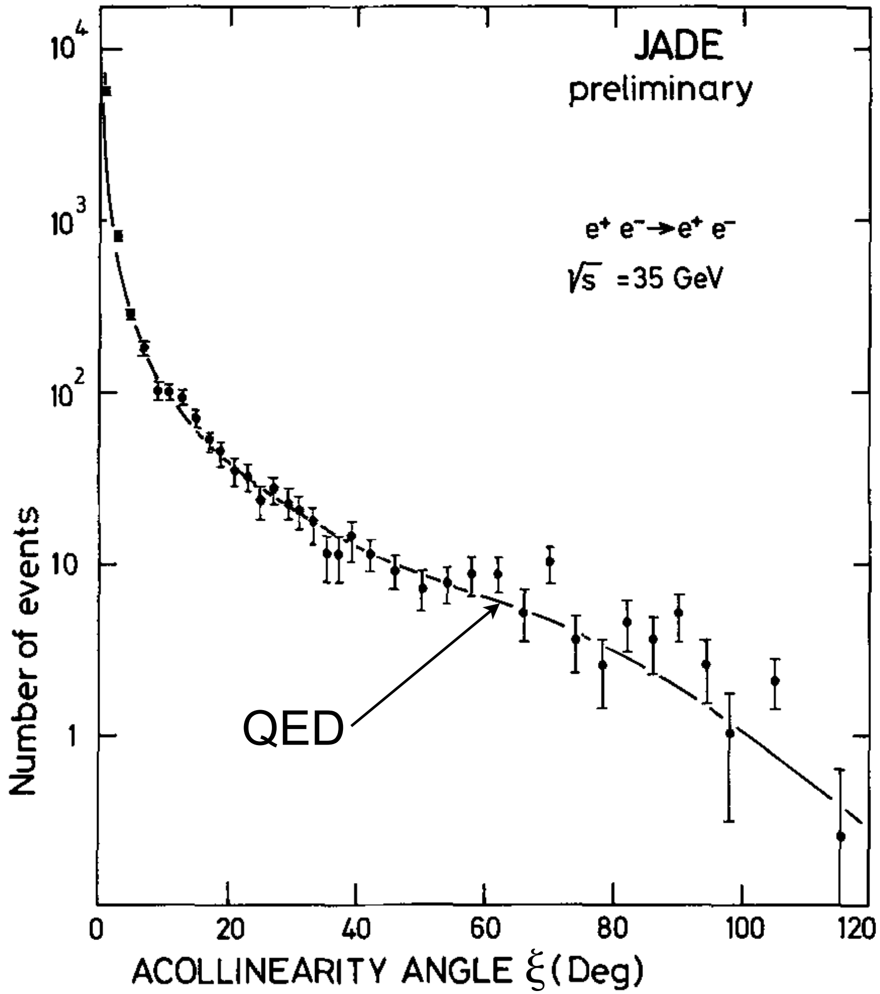


Initial (a) and final (b) state radiations produce an *acollinearity* of the final state particles:
outgoing particles are not exactly back-to-back

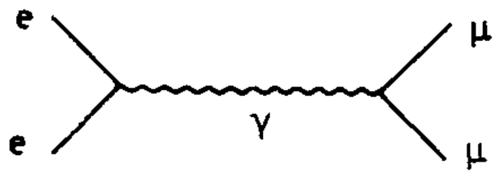
Acollinearity measurement



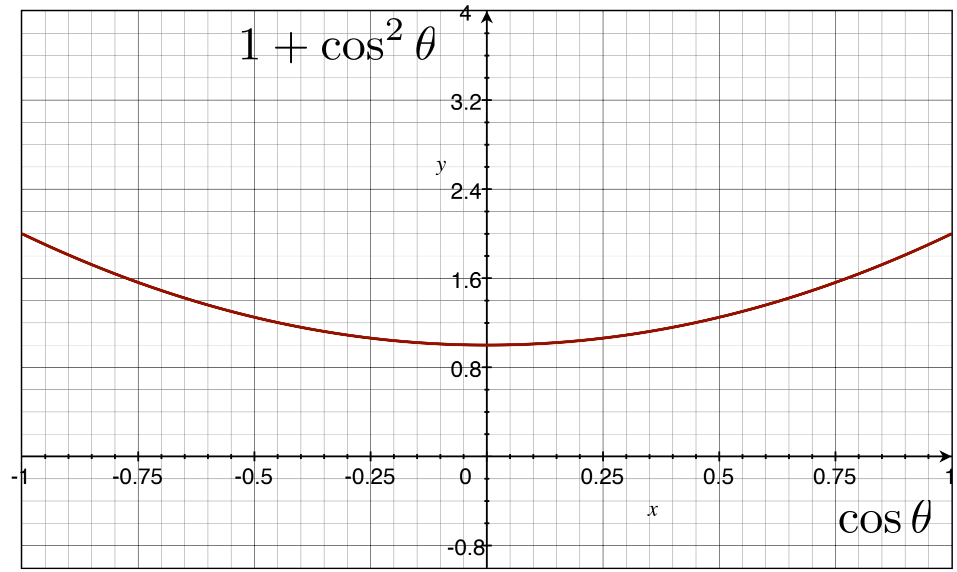
Radiative corrections calculated at α^3 order agree very well with the data



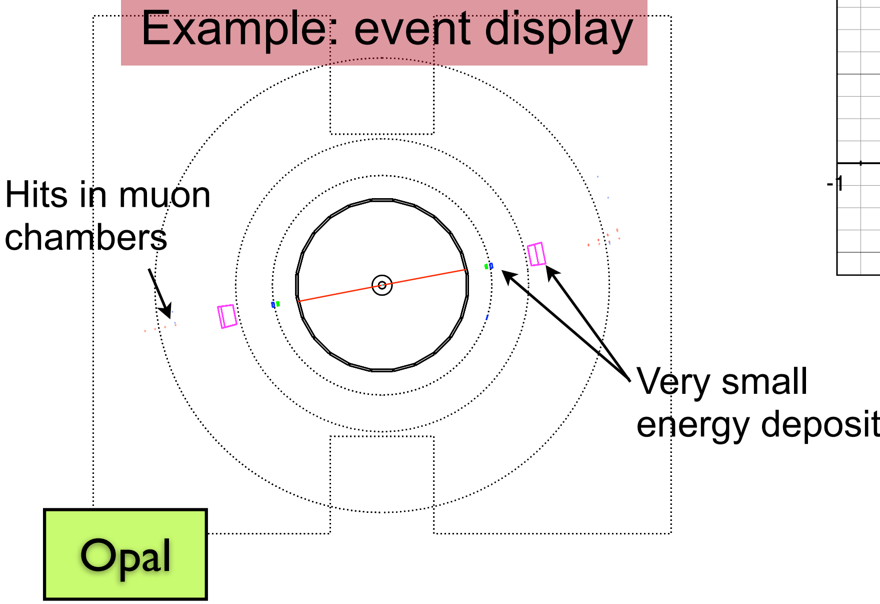
Muon pair production



$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{q'^4 + q^4}{s^2}\right) = \frac{\alpha^2}{2s}(1+\cos^2\theta)$$

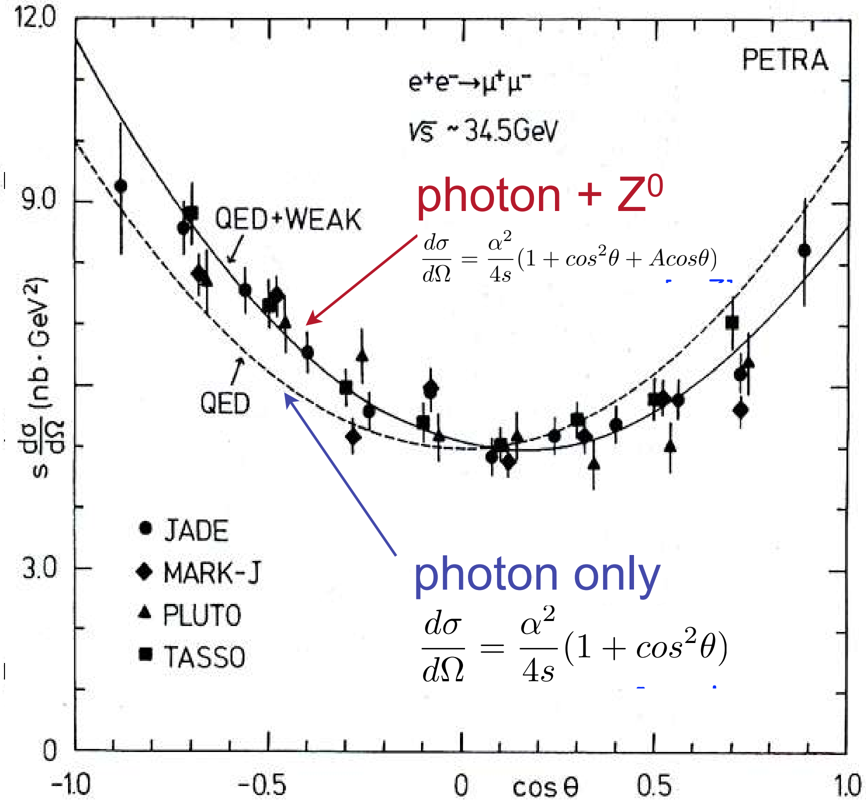


Example: event display

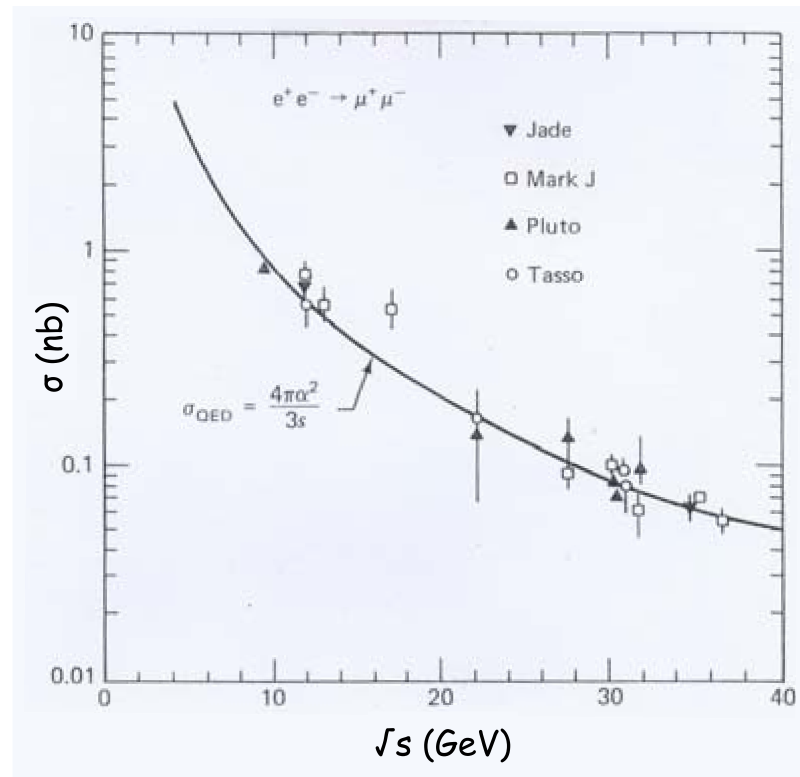


Muon pair production

Differential cross section

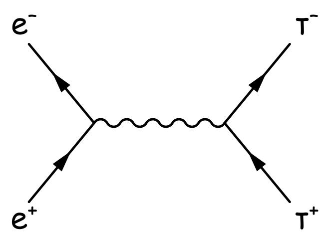


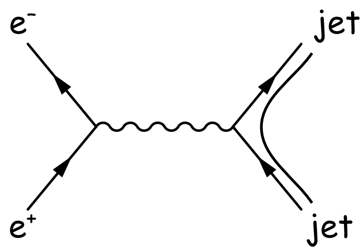
Total cross section vs. energy

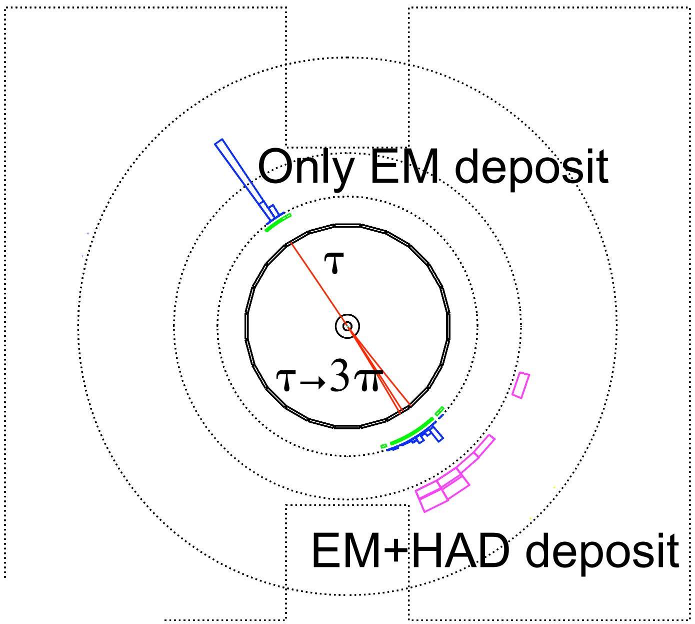


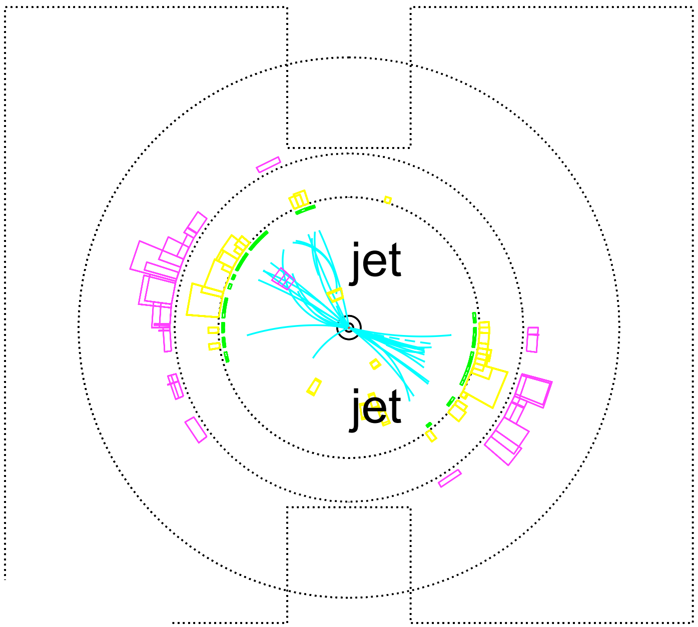
- Angular distribution is sensitive to electroweak corrections due to Z^0 exchange
  - Additional term proportional to $\cos(\theta)$
- Total cross section (integrating over solid angle) goes as $1/s$

Other final states



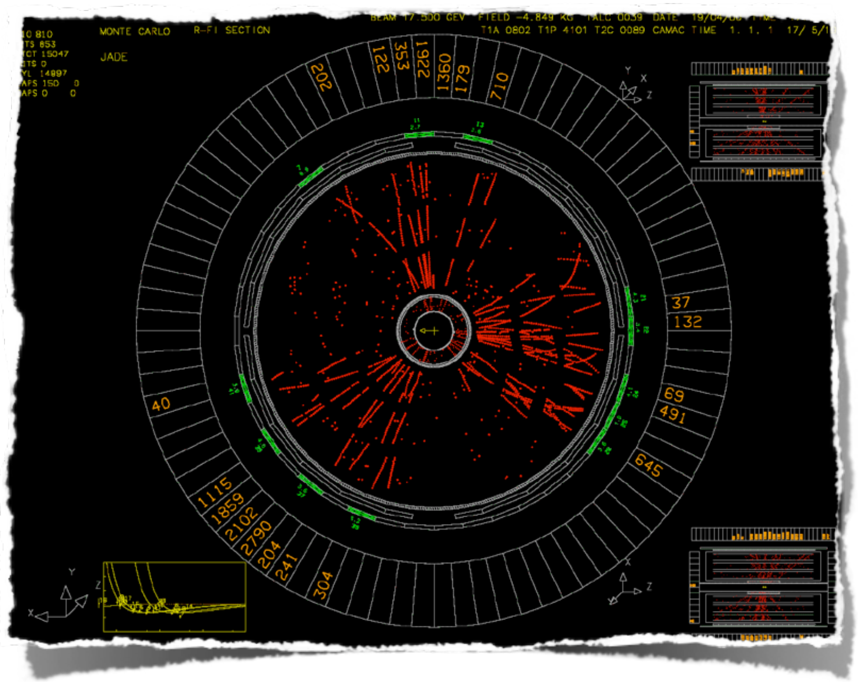


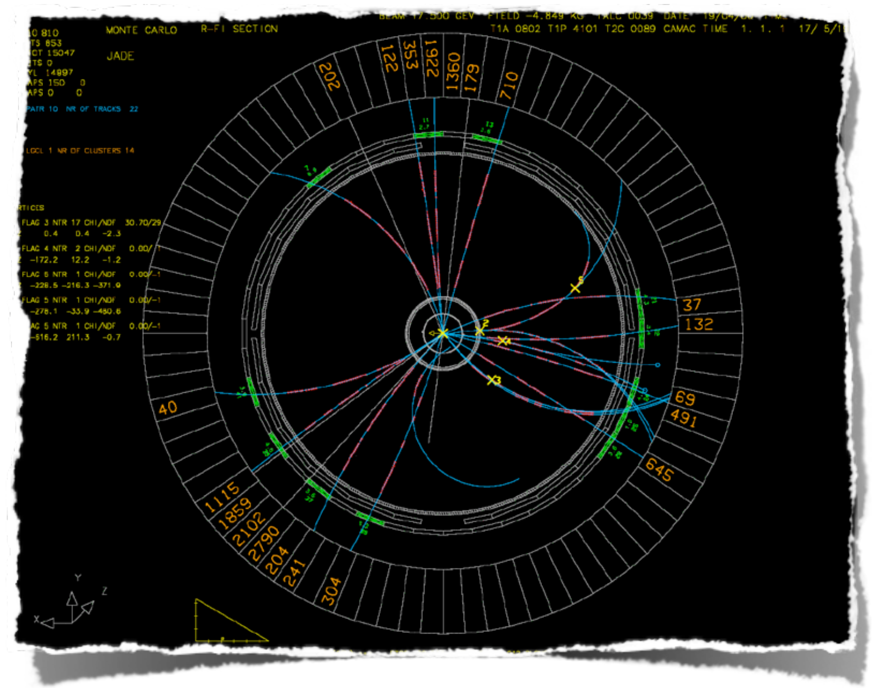




3 jets final states

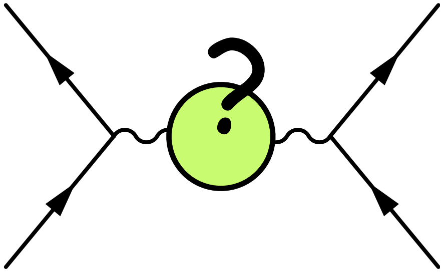
Hard gluon radiation from final quark-antiquark pair





Limits of QED

Question: what do we expect if QED is not the only physics involved in the scattering processes discussed so far?



We define an energy scale Λ, or distance $r \sim 1/\Lambda$, at which the QED theoretical model does not describe the data

Change of potential

$$\frac{1}{r} \rightarrow \frac{1}{r}(1 - e^{-\Lambda r})$$

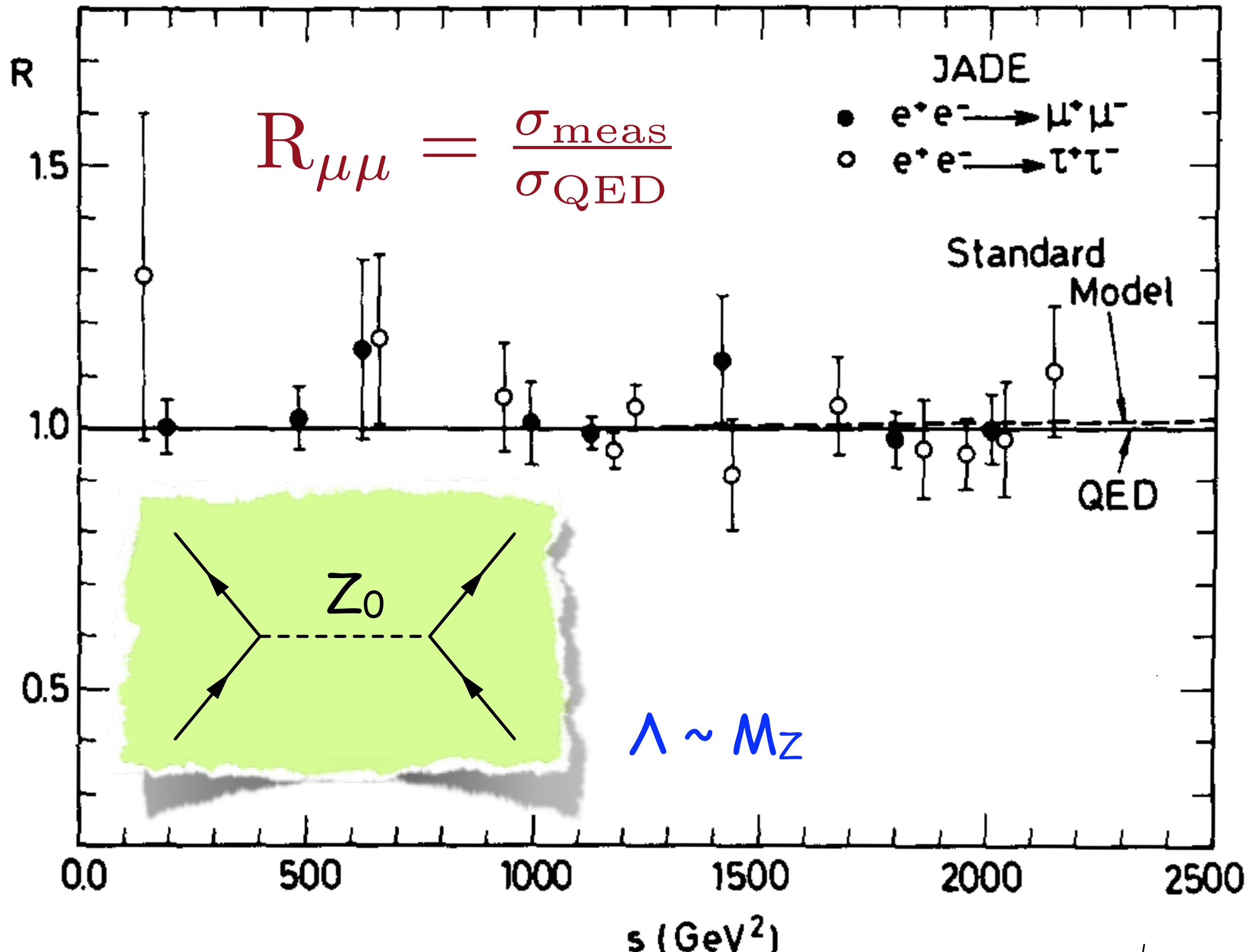
Change of propagator

$$-\frac{1}{q^2} \rightarrow -\frac{1}{q^2}\left(1 + \frac{q^2}{\Lambda^2}\right)$$

Change of cross section

$$\sigma_{\mu\mu} = \frac{3\pi\alpha^2}{3s}\left(1 \pm \frac{s}{\Lambda^2 - s}\right)^2$$

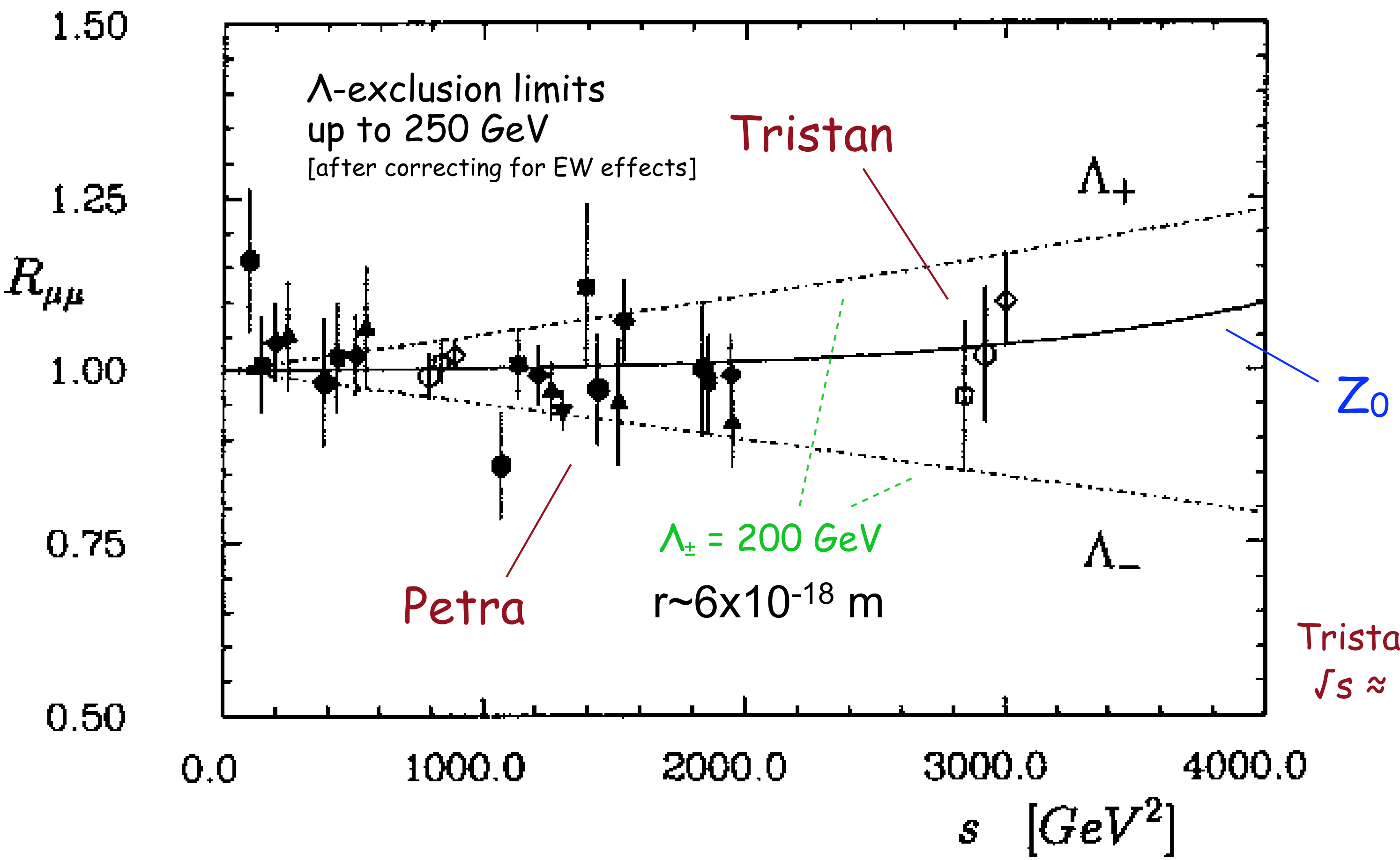
Beyond QED



$$R_{\mu\mu} = \frac{\sigma_{\text{meas}}}{\sigma_{\text{QED}}}$$

PETRA $\sqrt{s}$ < 47 GeV

Beyond QED



Tristan:
$\sqrt{s} \approx 55$ GeV

Part two:
Measurement of the electron anomalous magnetic moment

Electron magnetic moment

- Rotating electrically charge body creates a magnetic dipole
  - External magnetic field exerts a torque on the electron magnetic moment
- Electrons have intrinsic **magnetic moment**, related to their spin

$$\mu = -g\frac{e}{2m}s = -\frac{g}{2}\frac{e}{2m}$$

- In case of electrons the magnetic moment is **anti-parallel** to the spin
- The **g-factor is equal to 2**, as calculated from Dirac's equation

$$a = \frac{\alpha}{2\pi} = \frac{g-2}{2} = 0$$

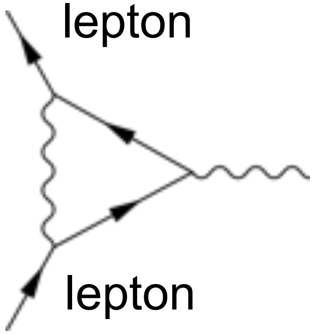
- Corrections to the g-factor are given by **higher order QED contributions** as well as **hadronic** and **weak interactions**. There could be additional contributions from **physics beyond the Standard Model (SM)**

$$\frac{g}{2} = 1 + a_{QED}(\alpha) + a_{hadronic} + a_{weak} + a_{new}$$

- **When adding the corrections we usually talk of <u>anomalous</u> magnetic moment of the electron**

QED: higher order corrections

- The one-loop corrections to the magnetic moment are due to vacuum fluctuation and polarization effects. For example:

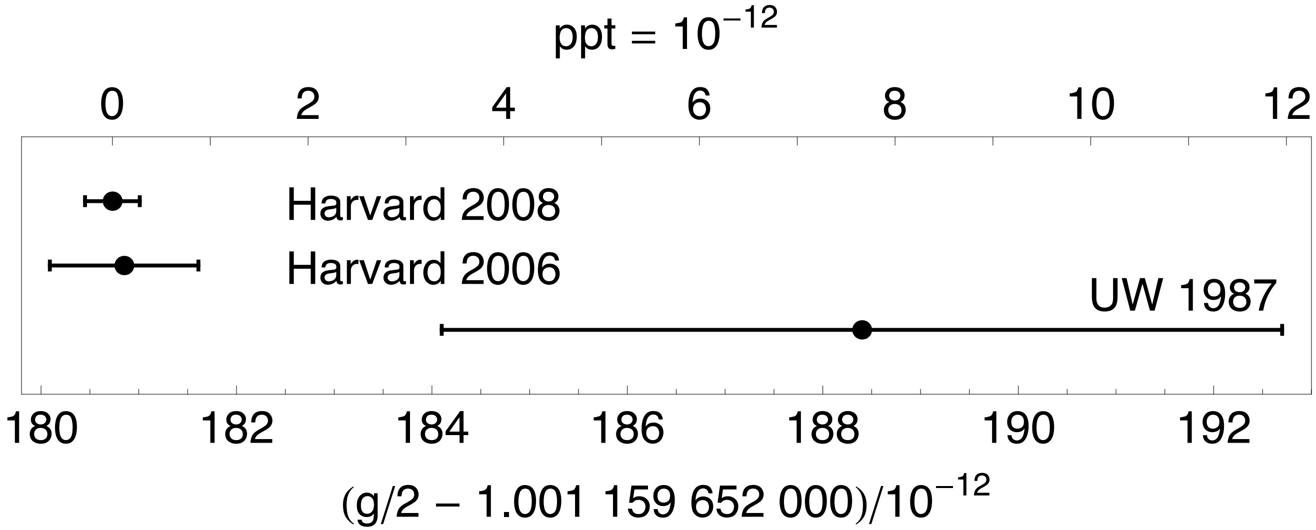


- The textbook calculation of the one-loop corrections gives corrections ~ 10^{-3} (see References):

$$a = \frac{\alpha}{2\pi} = \frac{g-2}{2} \simeq .0011614$$

- Hadronic and weak interactions are calculated (within the SM) to be very small and negligible, respectively
- As we will see, the precision achieved by experimental results need QED predictions with α^4 **precision**

Current status of g/2 measurements



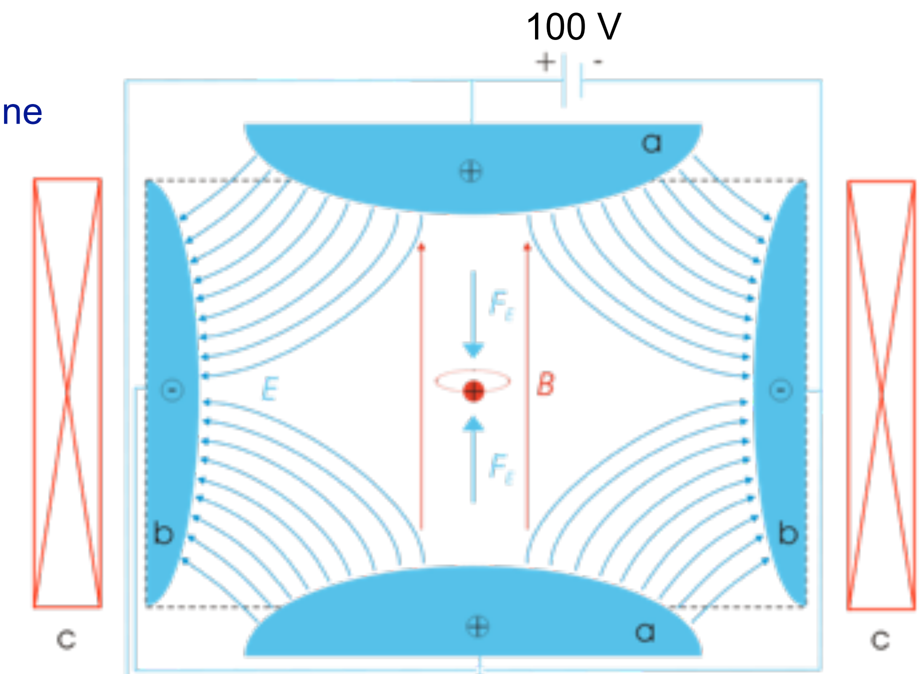
- The precision nowadays is below 10^{-12}!
- Latest measurements 15 times more precise than previous result, which stood for about 20 years
- Measured value is shifted by 1.7 standard deviations
- How did we get to this astonishing precision?

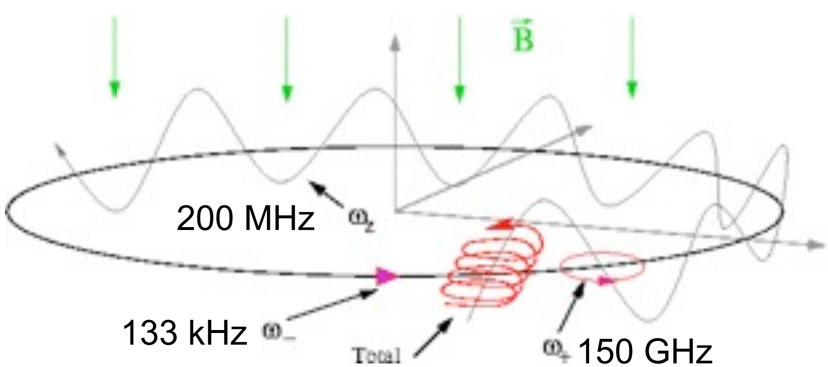
Experiment: main ingredients

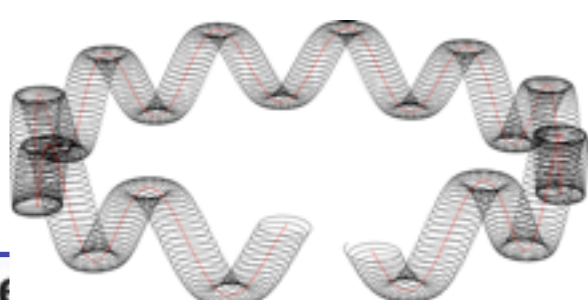
- Single-electron quantum cyclotron:
  - *A Penning trap suspends and confines the electron in an atom-like state*
- Fully resolved cyclotron and spin energy levels:
  - *Accurate measurements of the resonant frequencies of driven transitions between the energy levels of this homemade atom – an electron bound to our trap – reveals the electron magnetic moment in units of Bohr magnetons, g/2*
- Detection sensitivity sufficient to detect one quantum transitions
  - *Frequency detection sensitivity in the radio and microwave region*

Penning trap

- Penning trap confines electrons by using:
  - A strong **vertical magnetic field** to confine particles radially
  - A **quadrupole electric field** to confine particles axially
- The magnetic field is produced by a **solenoid**
- The electric field is produced by three electrodes: one **ring** and two **endcaps**
- The trajectory in the radial plane is characterized by two frequencies







Magnetron frequency: ω_-
Modified cyclotron frequency: ω_+
The cyclotron frequency is $(\omega_+ + \omega_-)$
A small-frequency oscillation is also in the vertical plane (axial frequency ω_z)

Energy levels measurement

- A non-relativistic electron in a magnetic field has energy levels:

$$E(n, m_s) = \frac{g}{2} h\nu_c m_s + \left(n + \frac{1}{2}\right) h\nu_c \qquad \nu_c = \frac{eB}{2\pi m} \qquad \nu_s = \frac{g}{2}\nu_c = \frac{g}{2}\frac{eB}{2\pi m}$$

- Depend on the **cyclotron frequency** (ν_c) and on the **spin frequency** (ν_s)

$$\frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c} = 1 + \frac{\nu_a}{\nu_c}$$

- Since ν_s and ν_c differ only by a part per 10^3 measuring ν_a and ν_c to a precision of 1 part on 10^{10} gives $g/2$ to **1 part to 10^{13}**.
- Two advantages of this technique:
  - **One can measure the ratio of two frequency to very high precision**
  - **Since the B field appears in both numerator and denominator, the dependence on the magnetic field cancels in the ratio**

Energy levels measurement

- Including the relativistic corrections, the energy levels are given by:

Relativistic correction term

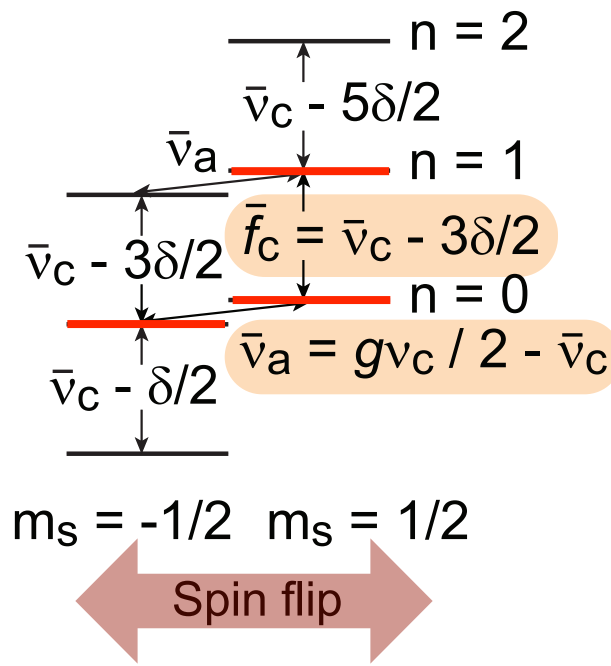
Frequency shift due to Penning trap

$$E(n, m_s) = \frac{g}{2} h\nu_c m_s + (n + \tfrac{1}{2})h\bar{\nu}_c - \tfrac{1}{2}h\delta(n + \tfrac{1}{2} + m_s)^2$$

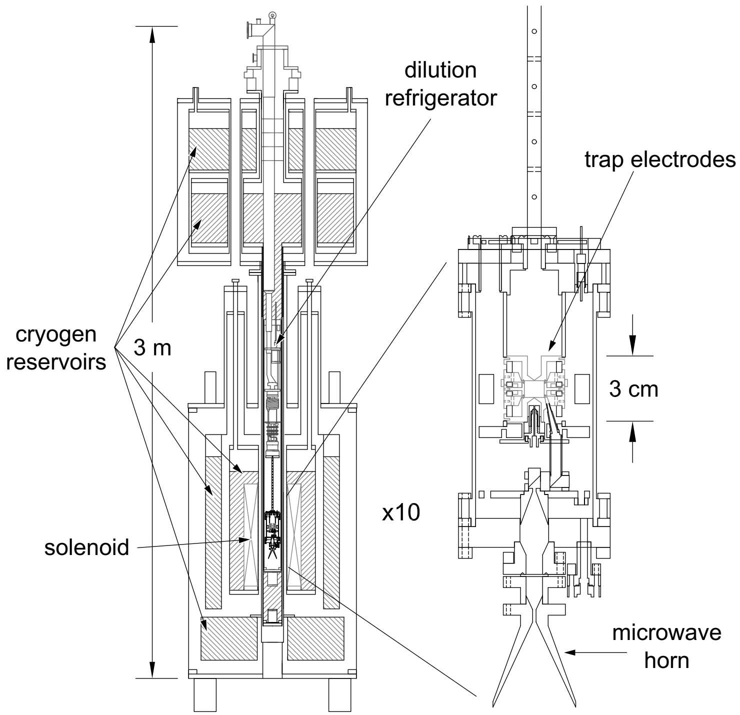
- The experiment measures the following transition frequencies:

$\bar{f}_c \equiv \bar{\nu}_c - \frac{3}{2}\delta$	1,1/2 → 0,1/2
$\bar{\nu}_a \equiv \frac{g}{2}\nu_c - \bar{\nu}_c$	0,1/2 → 0,-1/2

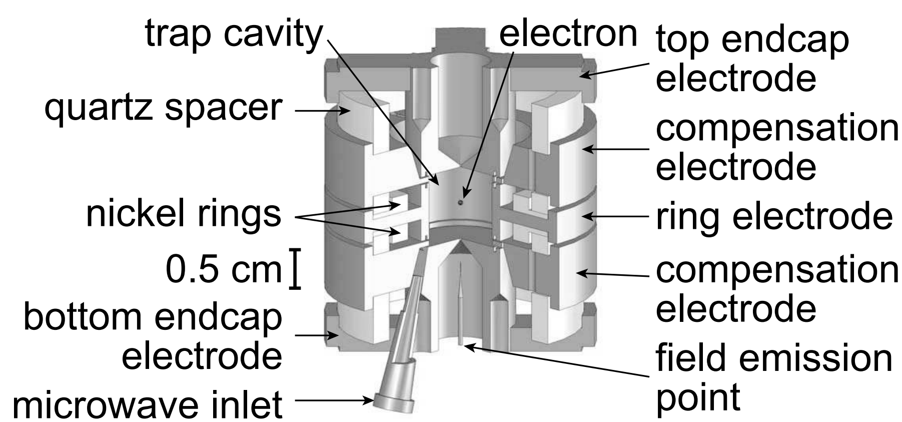
Cyclotron frequency ~ 150 GHz



Experimental setup



- A Penning trap is used to artificially bound the electron in an orbital state
- High voltage (100V) applied between cylindric and endcap contacts

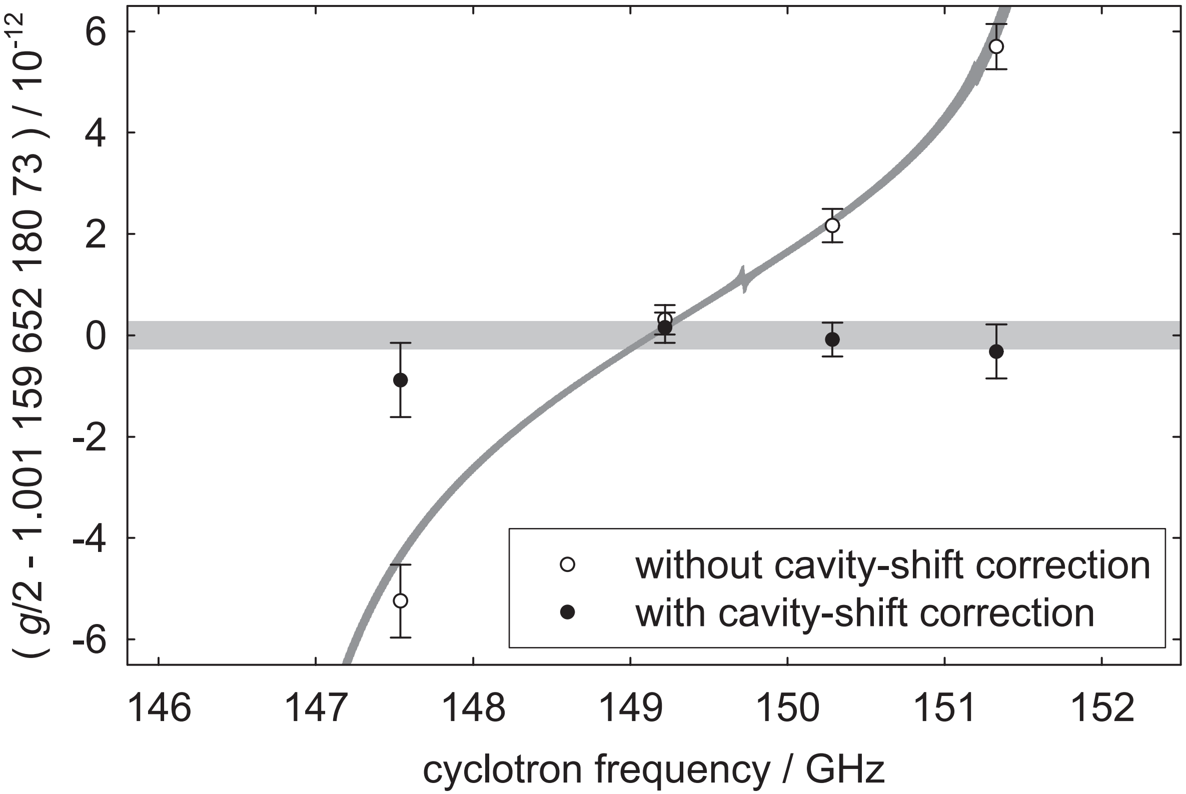


- A high magnetic field (5 T) is necessary to increase the spacing between cyclotron energy levels ($\nu_c \propto B$)
- Very low temperature (100 mK) increases the probability to populate the orbital ground state

$$P \propto \exp[-h\bar{\nu}_c/(kT)]$$

Results

$$g/2 = 1.001\,159\,652\,180\,73\,(28) \qquad [0.28 \text{ ppt}]$$



Shifts are induced by interaction of electron with nearby cavity radiation modes

Solution: do measurements at various frequencies

Theoretical predictions

- The QED calculations provide the prediction for g/2 up to the fourth power of alpha:

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\text{hadronic}} + a_{\text{weak}},$$

$$\begin{aligned} C_2 &= 0.500\ 000\ 000\ 000\ 00\ (\text{exact}) \\ C_4 &= -\,0.328\ 478\ 444\ 002\ 90\ (60) \\ C_6 &= 1.181\ 234\ 016\ 827\ (19) \\ C_8 &= -\,1.914\ 4\ (35) \\ C_{10} &= 0.0\ (4.6). \\ a_e^{\text{hadronic}} &= 1.682(20) \times 10^{-12} \end{aligned}$$

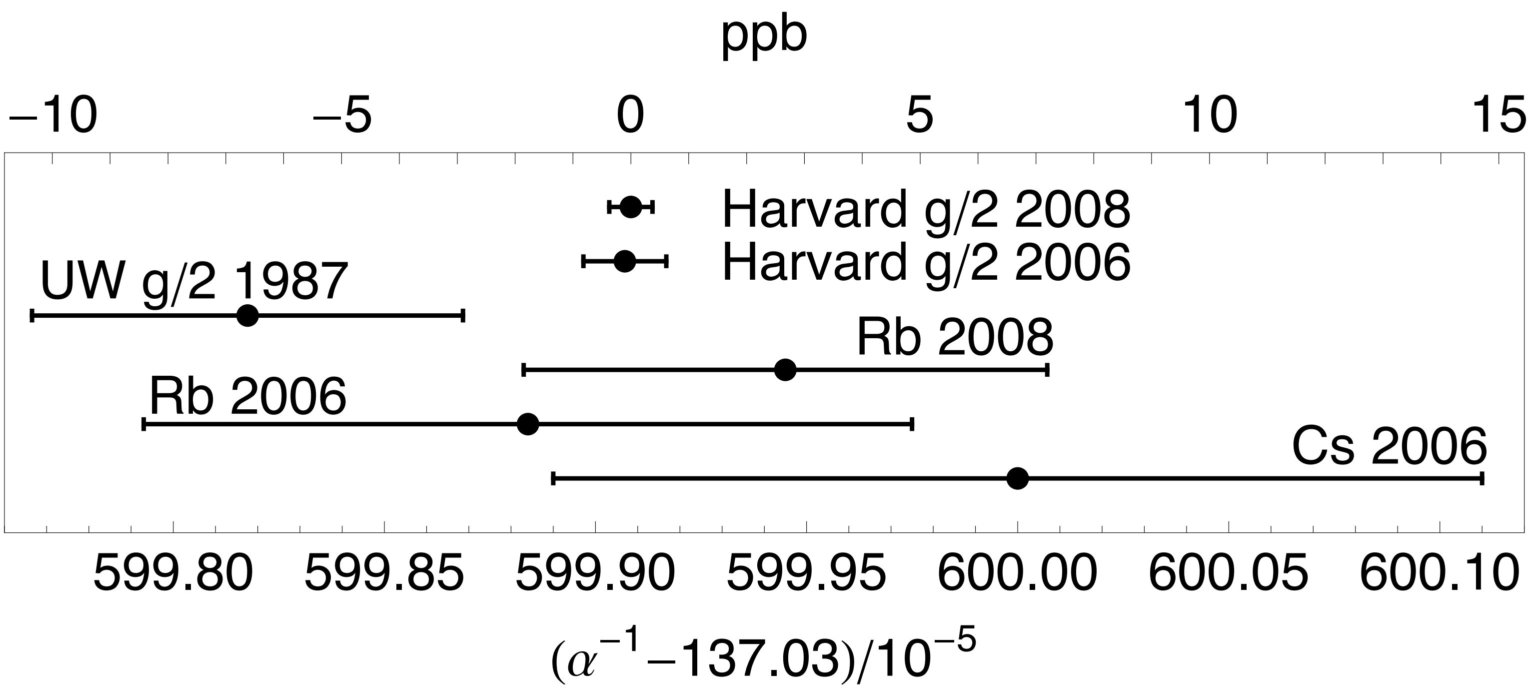
- From this formula and theoretical predictions we can:
  - Measure the coupling constant α

$$\begin{aligned} \alpha^{-1} &= 137.035\,999\,084\,(33)\,(39) \quad [0.24\ \text{ppb}]\,[0.28\ \text{ppb}] \\ &= 137.035\,999\,084\,(51) \quad [0.37\ \text{ppb}]. \end{aligned}$$

  - Comparing the measured $g/2$ with expectation using α from other measurements

$$\begin{aligned} g/2 &= 1.001\ 159\ 652\ 180\ 73\ \ (28)\ [0.28\ \text{ppt}], && \textit{Measured} \\ g(\alpha)/2 &= 1.001\ 159\ 652\ 177\ 60\ (520)\ [5.2\ \ \text{ppt}]. && \textit{Predicted} \end{aligned}$$

Status of high-precision α measurements



Source: http://hussle.harvard.edu/~gabrielse/gabrielse/papers/2009/DeterminingTheFineStructureConstant.pdf

References

- General reference for theory:
  - R.Feynman, *QED: The Strange Theory Of Light And Matter*, Princeton University Press, 1988
- Calculation of electron magnetic moment at one-loop precision:
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