Particle Physics Phenomenology I

HS 10, Series 3

Due date: 15.10.2010, 1 pm

Exercise 1 Consider an unpolarised decay of one particle to three new particles

$$p \to p_1 + p_2 + p_3,$$

e.g. the β-decay of a muon ($\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$). Calculate the three-particle phase space R_3 in the rest frame of the decaying particle, assuming $m_\mu \gg m_{e,\nu}$, i.e. show

$$R_3 = \pi^2 \int_0^{\frac{m_\mu}{2}} \mathrm{d}E_1 \int_{\frac{m_\mu}{2}-E_1}^{\frac{m_\mu}{2}} \mathrm{d}E_3. \tag{1}$$

Exercise 2

Show that the unitarity of the S-matrix ($SS^+ = 1$) together with $S_{fi} = \delta_{fi} + iT_{fi}(2\pi)^4\delta(p_f - p_i)$ implies the following:

(i) $T_{fi} - T^*_{if} = i(2\pi)^4 \sum_n \delta(p_f - p_n) T_{fn} T^*_{in}$

(ii) for $i = f$, means elastic forward scattering ($\Theta = 0$) with two particles a and b in the initial state i,

$$\mathrm{Im}\, M_{ii} = \sqrt{\lambda(s, m_a^2, m_b^2)}\, \sigma_{\mathrm{tot}} \qquad \text{(the optical theorem)} \tag{2}$$