Particle Physics Phenomenology I

HS 10, Series 6

Due date: 05.11.2010, 1 pm

Exercise 1 Show that if we postulate the commutation relations

$$\left[a(\vec{p}), a(\vec{q})^\dagger\right] = \delta^3(\vec{p}-\vec{q})(2\pi)^3 2E_{\vec{p}}$$
$$[a(\vec{p}), a(\vec{q})] = \left[a(\vec{p})^\dagger, a(\vec{q})^\dagger\right] = 0$$

for a and $a^\dagger$ we arrive at the following commutation relations for the field and the canonical momentum density conjugate to it

$$\left[\phi(\vec{x},t), \Pi(\vec{x'},t)\right] = i\delta^3(\vec{x}-\vec{x'})$$
$$\left[\phi(\vec{x},t), \phi(\vec{x'},t)\right] = \left[\Pi(\vec{x},t), \Pi(\vec{x'},t)\right] = 0.$$

Exercise 2 We define

$$\Delta^\pm(x) = -\int_{C^\pm} \frac{dp^0}{2\pi} \int \frac{d^3p}{(2\pi)^3} \frac{e^{-ipx}}{p^2-m^2}$$

where C^+ and C^- are contours in the complex p^0-plane, C^+ goes around $p^0 = E_{\vec{p}}$ once in counterclockwise direction, C^- goes around $p^0 = -E_{\vec{p}}$ once in counterclockwise direction.

- Use the residue theorem and the formula

  $$\int \frac{d^3p}{2E_{\vec{p}}} = \int d^4p\,\delta(p^2-m^2)\Theta(p^0)$$

  to show

  $$\Delta^\pm(x) = \mp i \int \frac{d^4p}{(2\pi)^3}\delta(p^2-m^2)e^{\mp ipx}\Theta(p^0).$$

- Show that $[\phi(x), \phi(y)] = i\Delta(x-y) = i(\Delta^+(x-y) + \Delta^-(x-y))$ vanishes for spacelike $((x-y)^2 < 0)$ separation $x-y$.