Particle Physics Phenomenology I

HS 10, Series 8

Due date: 26.11.2010, 1 pm

Exercise 1

The aim of this exercise is to evaluate the tree-level cross section for the process $e^+(p_1) + e^-(p_2) \to \gamma(k_1) + \gamma(k_2)$ in the high energy limit, $s \gg m_e^2$.

(i) Apply crossing relations to the Compton Scattering amplitude to show that the spin averaged squared amplitude is given by

$$\frac{1}{4}\sum_{spins} |M_{e^+e^-\to\gamma\gamma}|^2 = 2e^4\left(\frac{u}{t}+\frac{t}{u}\right).$$

(ii) Show that the differential cross section element for this process is given by

$$d\sigma_{e^+e^-\to\gamma\gamma} = \frac{1}{2s}\frac{d\cos\theta}{16\pi}|M_{e^+e^-\to\gamma\gamma}|^2.$$

where θ can be taken to be the angle between p_1 and k_1.

(iii) Parameterise u and t in terms of $\cos\theta$ and show that the cros section becomes

$$\sigma_{e^+e^-\to\gamma\gamma} = \frac{e^4}{16\pi s}\int_{-1}^{1} d\cos\theta\left[\frac{1+\cos\theta}{1-\cos\theta}+\frac{1-\cos\theta}{1+\cos\theta}\right].$$

(iv) Conclude that the above integral has divergences at $\cos\theta = \pm 1$,i.e. whenever the photons are collinear to the beam. Argue that in those limits the photons are undetectable and that we can therefore regularise the integral with a small (experimental!) cutoff, such that $-1+\epsilon < \cos\theta < 1-\epsilon$. Hence derive

$$\sigma_{e^+e^-\to\gamma\gamma} = \frac{e^4}{4\pi s}\left(-1+\epsilon+\ln\left(\frac{2-\epsilon}{\epsilon}\right)\right).$$

(v) Plot your result for $\epsilon \in (0,1] \subset \mathbb{R}$ keeping other constants one. Is it always positive? What angular region does this correspond to?

(vi) Argue or show that the Amplitude does not diverge if $m_e > 0$.

Exercise 2 Use the feynman rule for the quark anti-quark photon vertex to infer that the tree level cross section for $q\bar{q} \to \gamma\gamma$ should be related to the tree level cross section for $e^+e^- \to \gamma\gamma$ by

$$\sigma_{q\bar{q}\to\gamma\gamma} = \left(\sum_{f=1}^{N_f} Q_f^4\right) \frac{\sigma_{e^+e^-\to\gamma\gamma}}{N_c}.$$

Q_f shall denote the fractional electric charge of the flavour of the quark -antiquark pair ($q\bar{q}$), N_f the number of different quark-flavours and N_c the number of different colors.

Exercise 3

Consider the event $pp \to \mu^+\mu^-\mu^+\mu^-$ (p denotes a Proton) with the total invariant mass of the muons suming to $(185Ge)^2$.

(i) Could the muons have been produced by two Z-bosonsif or off-shell photons ($k_1^2 = M_z^2 = k_2^2$) which were produced by a $q\bar{q}$-pair (or a Higgs boson)?

(ii) Could these muons also have been produced by two on-shell ($k_1^2 = 0 = k_2^2$) photons (which at leading order were produced by a $q\bar{q}$-pair or even a Higgs)?

(iii) If so what could you say about the energies and transverse momenta of the photons?

(iv) What could you say about the minimum mass the Higgs boson could have had?