Quantum Field Theory III

HS 10, Exercise sheet 11

Due date: 08.12.2010

Exercise 1:
In $N = 2$ supersymmetric theories we have not only vector multiplets but also hypermultiplets. A hypermultiplet is built by taking two chiral superfields[1]

$$\begin{aligned} H_1 &= (H^+, \eta_\alpha^+, F^+), \\ H_2 &= (H^-, \eta_\alpha^-, F^-). \end{aligned}$$

The scalar components H^+ and H^- form an $SU(2)$ doublet, while the other components are $SU(2)$ singlets.

a) Write down the most general renormalizable supersymmetric Lagrangian with a discrete R-symmetry

$$\begin{aligned} H^+ &\rightarrow -(H^-)^\star, \\ H^- &\rightarrow (H^+)^\star \end{aligned}$$

and without gauge interactions.

b) In $N = 2$ supersymmetry we can introduce a gauge theory by combining two $N = 1$ supermultiplets, a vectormultiplet $V^A = (v_\mu^A, \lambda^A, D^A)$ and a chiral multiplet $\Phi^A = (Z^A, \psi^A, F^A)$, and imposing a discrete R-symmetry

$$\psi^A \rightarrow \lambda^A \quad \text{and} \quad \lambda^A \rightarrow -\psi^A.$$

The Yang-Mills Lagrangian for this theory reads

$$\mathcal{L}_{YM}^{N=2} = \frac{1}{32\pi}\Im\left(\tau \int d^2\theta Tr W^\alpha W_\alpha + h.c.\right) + \int d^2\theta d^2\bar{\theta} Tr \bar{\Phi} e^{2gV} \Phi$$

How can you impose a gauge theory on the hypermultiplet Lagrangian in exercise 1a)?

c) Why does $N = 2$ supersymmetry require H_1 and H_2 to be in a complex conjugate representation of the gauge group?
Hint: You can consider the easy case of an $U(1)$ gauge theory and then generalize to any gauge theory.

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[1] Here, the plus superscripts, e.g. H^+ denote the components of H_1. Complex conjugates are denoted by $\bar{H}^+$.