Quantum Field Theory III

HS 10, Exercise sheet 8

Due date: 17.11.2010

Exercise 1:
Compute the non-Abelian generalization of the field strength superfield

$$W_\alpha = -\frac{1}{8}(\bar{D}\bar{D})e^{-2V}D_\alpha e^{2V}$$

in Wess-Zumino gauge expressed in $y, \theta, \bar{\theta}$ - coordinates

$$V = V_{WZ} = \theta\sigma^\mu\bar{\theta}v_\mu(y) + i(\theta\theta)\bar{\theta}\bar{\lambda}(y) - i(\bar{\theta}\bar{\theta})\theta\lambda(y) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})\left(D(y) - i\partial_\mu v^\mu(y)\right).$$

Here it is understood that all fields are contracted with the hermitian generators, i.e. $V \equiv V^A t^A$, implying $v_\mu \equiv v_\mu^A t^A$, $\lambda \equiv \lambda^A t^A$ and $D \equiv D^A t^A$.

Exercise 2:
Consider the Lagrangian of supersymmetric QED

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\left(\bar{\Phi}_+ e^{2eV}\Phi_+ + \bar{\Phi}_- e^{-2eV}\Phi_-\right) + \left[\int d^2\theta\left(\frac{1}{4}W^\alpha W_\alpha + m\Phi_+\Phi_-\right) + h.c.\right].$$

This Lagrangian is invariant under $U(1)$ gauge transformations

$$\begin{aligned} \Phi_\pm &\rightarrow e^{\pm ie\Lambda}\Phi_\pm \\ V &\rightarrow V - \frac{i}{2}(\Lambda - \bar{\Lambda}) \\ W^\alpha &\rightarrow W^\alpha, \end{aligned}$$

where $\pm e$ is the charge of the chiral superfield $\Phi_\pm$[1].
Expand this Lagrangian in components and verify that it describes a massless gauge boson (the photon) and a charged massive fermion (the electron), as well as a massless neutral fermion (the photino) and a massive charged scalar (the selectron).

Hint: Use the gauge invariance of the Lagrangian to perform the calculation in the Wess-Zumino gauge.

[1] Note that we need two oppositely charged chiral superfields Φ_+ and Φ_- to have a gauge invariant massterm. The corresponding massless theory is invariant with only one chiral superfield Φ.