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# EFFECTIVE NONRENORMALIZABLE THEORIES AT ONE LOOP*

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MASTER

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# 1. Effective Nonrenormalizable Theories in Physics

## 1.1. Infrared limits of the standard model.

Observed particle interactions can sometimes be described by effective nonrenormalizable theories that, in the context of the standard model for strong and electroweak interactions, correspond to a particular long distance, or low energy, limit of the underlying renormalizable theory. For example, the Fermi theory of $\beta$-decay correctly describes weak charged current interactions in the limit of small momentum transfer $q_\mu$, compared with the mass $m_W$ of the charged intermediate bosons $W^\pm$ that mediate these interactions:

$$|q^2| << m_W^2 = \sqrt{2}g^2/8G_F, \tag{1.1}$$

where $G_F$ is the Fermi coupling constant. Another example is the $SU(2)_L \times SU(2)_R$ chiral invariant $\sigma$-model that describes pion dynamics at energies that are small compared with the inverse confinement radius of QCD. However, in this case, we cannot simply reproduce the effective low energy theory as a particular limit of a parameter (e.g., $m_W \to \infty$ for the electroweak theory) of the QCD Lagrangian; numerical methods used in attempts to establish such a connection will be described in the lectures of Petronzio [1].

Finally, quantum gravity and its supersymmetric extension, supergravity, are nonrenormalizable theories that are often conjectured to be the low energy/long distance limit of a finite (rather than renormalizable) theory which should become manifest at energy scales large compared to the Planck scale or some other mass parameter characterizing the underlying physics. The current leading candidate for such a theory is a superstring theory[2] in ten dimensions, in which case the relevant parameter could be the compactification scale or the string tension, both of which are expected to be within a few orders of magnitude of the Planck mass.

Effective four dimensional field theories suggested by superstring theories generally have a high degree of vacuum degeneracy at tree level which is related to symmetries of the effective Lagrangian under nonlinear transformations among scalar fields, similar to the chiral invariance of the nonlinear $\sigma$-model for low energy pions. An important question then is to what degree the degeneracy is lifted by loop corrections to the effective tree Lagrangian. In this lecture I will discuss one-loop corrections to effective nonrenormalizable theories, with special attention to loop expansion techniques that preserve all the invariances of the effective tree Lagrangian. Such symmetries play an important role in the superstring-inspired field theories that I will discuss in my second lecture. Here I illustrate the relevant techniques with examples drawn from the standard model where it is possible to compare results using the effective low energy/long distance nonrenormalizable theory with exact calculations in the underlying renormalizable theory.

Recall first two important properties of ultraviolet divergent contributions at each order in the loop expansion for renormalizable theories: a) They are at most logarithmic - with the important exception of quadratically divergent contributions to scalar masses that I will discuss later in relation to the gauge hierarchy problem. b) They can be reabsorbed into redefinitions of the parameters of the tree Lagrangian - coupling constant, fermion masses, etc.

Now consider the Fermi theory of low energy charged current weak interactions.

The effective interaction tree Lagrangian is of the form:

$$\mathcal{L}_{tree} = 2\sqrt{2}G_F(\bar{\psi}_L\gamma^\mu\psi_L)(\bar{\psi}_L\gamma_\mu\psi_L). \tag{1.2}$$

The one-loop contribution, Fig. 1a, to the effective four-fermion coupling is quadratically divergent. Cutting off the loop momentum integration at $|p| = \Lambda$ gives the

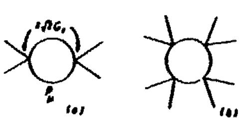


**Figure 1: Divergent one-loop contributions to 4-point (a) and 8-point (b) functions in the Fermi theory.**

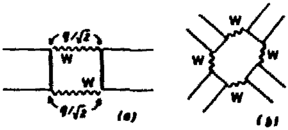


**Figure 2: Finite one-loop contributions to 4-point (a) and 8-point (b) fermion functions in the renormalizable gauge theory.**

estimate (recall: there is a factor $(4\pi)^{-2}$ for each loop integration):

$$\mathcal{L}_{eff}^{1-loop} \sim \frac{1}{16\pi^2}\Lambda^2(2\sqrt{2}G_F)^2(\bar{\psi}_L\gamma\psi_L)^2 \sim \frac{G_F^2}{2\pi^2}\Lambda^2(\bar{\psi}_L\gamma\psi_L)^2. \tag{1.3}$$

In the context of the standard model, we know that the Fermi theory is relevant only for momenta $|p|^2 << m_W^2$; if we identify the cut-off $\Lambda$ with the scale at which the Fermi theory ceases to be valid, $\Lambda^2 \sim m_W^2$, we obtain, using (1.1):

$$\mathcal{L}_{eff} \sim g^2\frac{\sqrt{2}G_F}{16\pi^2}(\bar{\psi}_L\gamma\psi_L)^2 = \frac{\alpha}{8\pi}\mathcal{L}_{tree} \tag{1.4}$$

where $\alpha \equiv g^2/4\pi$ is the weak "fine structure" constant. The result (1.4) can be compared with the low energy limit for external momenta of the diagram of Fig. 2a, which is finite and yields the same estimate:

$$\mathcal{L}_{eff}^{1-loop} \sim \frac{1}{16\pi^2}\left(\frac{g}{\sqrt{2}}\right)^4\frac{1}{m_W^2}(\bar{\psi}_L\gamma\psi_L)^2 \sim \frac{\alpha}{8\pi}(\mathcal{L}_{tree})_{Fermi}. \tag{1.5}$$

In the context of the Fermi theory, the quadratically divergent one loop correction (1.3) can be absorbed into a redefinition of the Fermi coupling constant. However, there are also logarithmically divergent contributions to the one-loop effective action that generate new couplings. For example, the contribution of Fig. 1b can be estimated as

$$\mathcal{L}_{eff} \sim \frac{1}{16\pi^2}(2\sqrt{2}G_F)^4\ln(\Lambda^2/\mu^2)(\bar{\psi}_L\gamma\psi_L)^4, \tag{1.6}$$

where $\mu$ is a fermion mass $m_f$ or an external momentum $|q_{ext}|$. Dimensional considerations and an analysis of the infrared behavior of the corresponding finite diagram of Fig. 2b gives the estimate

$$\mathcal{L}_{eff} \sim \frac{1}{16\pi^2}\left(\frac{g}{\sqrt{2}}\right)^6 \frac{1}{m_W^6} \ln(m_W^2/\mu^2)(\bar{\psi}_L\gamma\psi_L)^4, \tag{1.7}$$

which, using (1.1), is the same as (1.6) for $\Lambda^2 = m_W^2$. Note that while the underlying physics dictates that $\Lambda = O(m_W)$, we cannot in general set $\Lambda = m_W$ as an exact equality. Rather, we should set $\Lambda = \eta m_W$ with $\eta = O(1)$. The precise value of $\eta$ depends on the details of the way in which new physics - in this instance the finite range $r \sim m_W^{-1}$ of the weak interaction - enters to damp the apparent divergences of the effective low energy theory. Moreover, the value of $\eta$ can differ from one diagram to another. Thus, calculations using the effective nonrenormalizable theory should reproduce the correct order of magnitude of the quadratically divergent terms as well as the precise coefficient of the logarithmic divergence. In the latter case a rescaling of $\Lambda$ by a factor of order unity can be reabsorbed into residual finite terms that cannot be reliably evaluated in the context of the effective theory.

The above analysis is appropriate for the Fermi theory of charged current couplings with one generation of quarks. When $u \leftrightarrow s$ charged current couplings are included in the effective tree Lagrangian (1.2), one would grossly overestimate one-loop strangeness-changing neutral current transitions with the identification $\Lambda \sim m_W$. This is because there is a much lower threshold, $\Lambda \sim m_c$ (c=charm) where these transitions are damped by the GIM mechanism[3]. Comparison of calculations of this type with data provided an estimate[3] of the charmed quark mass before the underlying theory[4] was known. In other words, an analysis of the divergent loop contributions to a known effective theory can point to thresholds where that theory must be replaced by a more convergent one.

In the following I will focus on a nonrenormalizable theory that is more closely related to those suggested by superstrings, namely a gauged nonlinear $\sigma$-model, but one which can also be obtained analytically in a particular limit of a parameter ($m_H \to \infty$) of the standard, renormalizable electroweak theory. This will provide another laboratory for testing the validity of calculations using the effective theory. We will find (as for certain superstring inspired models to be discussed later) features similar to those for the Fermi theory: quadratic divergences can be reinterpreted as renormalizations, while new terms are generated at the level of logarithmic divergences. I will also introduce, in the context of more familiar physics, notions such as scalar metric, scalar curvature and nonlinear symmetries, that play an important role in formal aspects of string theories discussed by other lecturers.

### 1.2 The large Higgs mass limit of the standard electroweak model.

Neglecting gauge couplings, the scalar sector of the standard model[4] has the (renormalizable) Lagrangian

$$\mathcal{L}_H = \partial_\mu\varphi\partial^\mu\bar{\varphi} - \lambda(|\varphi|^2 - \frac{v^2}{2})^2 \tag{1.8}$$

which is invariant under the group $SO(4)$ or $SU(2) \times SU(2)$ of linear transformations among the four real scalar fields that parametrize the complex doublet $\varphi$:

$$\varphi \equiv \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} i\pi_1 + \pi_2 \\ \sigma - i\pi_3 \end{pmatrix}. \tag{1.9}$$

In terms of the component fields $(\pi, \sigma)$ Eq. (1.8) takes the standard form of the linear $\sigma$ model:

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - \frac{1}{4}\lambda(\sigma^2 + \pi^2 - v^2)^2. \tag{1.10}$$

A useful nonlinear formulation is obtained by making the field redefinition

$$\varphi = \frac{1}{\sqrt{2}} e^{i\vec{\theta}\cdot\vec{\tau}/v} \begin{pmatrix} 0 \\ \rho \end{pmatrix}. \tag{1.11}$$

In terms of the field variables $(\theta, \rho)$ the Lagrangian

$$\mathcal{L}_H = \mathcal{L}_{K.E.}(\theta, \rho) - \frac{1}{4}\lambda(\rho^2 - v^2)^2 \tag{1.12}$$

displays explicitly the decoupling at zero four-momentum of the massless Goldstone modes $\theta_i$, since these fields appear in (1.12) only through derivative couplings ($\mathcal{L}_{K.E.}$). *The theories (1.10) and (1.12) are equivalent and give identical S-matrices* as calculated by expanding about the vacuum defined by $< |\varphi| > \sqrt{2} = < \rho > = < \sigma > = v$, $\rho = \sigma + O(|\varphi| - v/\sqrt{2})^2$, $\theta_i = \pi_i + O(|\varphi| - v/\sqrt{2})^2$.

Instead of (1.8), the Lagrangian relevant for weak interaction physics is that of a gauged scalar sector, with the replacement

$$\partial_\mu \varphi \to D_\mu \varphi \equiv (\partial_\mu + iA_\mu)\varphi, \quad A_\mu = \frac{g}{2} T_a A_\mu^a \tag{1.13}$$

where the four $2 \times 2$ matrices $T_a$ represent the generators of $SU(2)_L \times U(1)$ on the scalar doublet $\varphi$, and $A_\mu^a$ are gauge fields. The gauged Lagrangian is invariant under the transformation

$$\varphi' = U(x)\varphi, \quad A'_\mu = UA_\mu U^{-1} + i\partial_\mu U U^{-1} :$$

$$\mathcal{L}(A, \varphi) = \mathcal{L}(A', \varphi') \equiv \mathcal{L}(A, \varphi'), \tag{1.14}$$

where in writing the last line of (1.14) we have relabelled the gauge field $A' \equiv A$. In other words we treat the transformed gauge fields as the gauge degrees of freedom. With the particular choice

$$U = e^{-i\vec{\theta}\cdot\vec{\tau}/v}, \quad \varphi' = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho \end{pmatrix} \tag{1.15}$$

we obtain the Lagrangian of the "unitary gauge"

$$\mathcal{L}_U = \mathcal{L}(A, \rho), \tag{1.16}$$

and we identify the physical Higgs scalar as $H = \rho - v$. Loop calculations are most easily done in a renormalizable gauge in which $\varphi$ is represented linearly, Eq. (1.9) and the unphysical scalar degrees of freedom $\pi_i$ appear in the Lagrangian:

$$\mathcal{L}_R = \mathcal{L}(A, \sigma, \pi), \quad H \equiv \sigma - v. \tag{1.17}$$

In either case the physical Higgs mass is extracted from the potential in (1.10) or (1.12) as:

$$m_H^2 = 2v^2\lambda. \tag{1.18}$$

The physical particles of the theory are the massless photon $\gamma$, the massive vector bosons $W^\pm$ and $Z$ and the Higgs particle $H$. The vector boson masses are extracted by expanding the covariant derivative (1.13) around the vacuum $|\varphi| = v/\sqrt{2}$; in terms of the linear field variables (1.9):

$$D_\mu\varphi D^\mu\bar{\varphi} = \frac{g^2v^2}{4}\left(W_\mu^+W^{\mu-} + \frac{1}{2\cos^2\theta_w}Z_\mu Z^\mu\right) + m_W W_\mu^+\partial^\mu\pi^- + \cdots \tag{1.19}$$

The vacuum expectation value (vev) $v$ is fixed by the experimental determination of the Fermi constant $G_F$ and the identification

$$v^2 = \frac{4m_W^2}{g^2} = (\sqrt{2}G_F)^{-1} \simeq (\frac{1}{4}TeV)^2. \tag{1.20}$$

Although the $\pi_i$ are not physical degrees of freedom of the theory, the relevance of the $\sigma$-model (1.10) or (1.12) to physics is through a theorem [6-8] which states that $S$-matrix elements including longitudinally polarized $W$'s and $Z$'s$(W_L, Z_L)$ can be calculated, up to corrections of order $m_W/E_W$ and $m_Z/E_Z$, by replacing $W_L^\pm$ and $Z_L$, respectively, by $\pi^\pm$ and $\pi^0$ as external particles and using the Feynman rules of a renormalizable gauge, i.e., using the Lagrangian (1.17). This result is intuitively plausible if one recalls that the physical, or unitary, gauge of Eqs. (1.15) and (1.16) was obtained by a transformation $W_\mu \to W_\mu' \simeq W_\mu + \partial_\mu\pi$ that introduces a longitudinal component $\partial_\mu\pi$ into the vector field. Alternatively, in an unphysical gauge, the last term in (1.19) introduces a mixing of $W_\mu$ with the longitudinal vector $\partial_\mu\pi$. In practice, calculations are usually performed in a renormalizable $R_\xi$ gauge[9] in which the gauge fixing term is arranged to precisely cancel the $W - \pi$ mixing term in (1.19). The Lagrangian is no longer manifestly gauge invariant, but is invariant under nonlinear BRS transformations[10] that are related to gauge transformations. The Ward identities of BRS invariance can be used[7,8] to derive the vector-scalar equivalence theorem stated above.

Now consider the limit $m_H \to \infty$. Since $v$ is fixed by experiment, Eq. (1.20), it follows from (1.18) that $\lambda \to \infty$, i.e., that scalar self-interactions become strong.[11,6] If the potential energy-density in (1.10) or (1.12) is to remain finite in this limit, $|\rho|^2$ must be fixed at its ground state value.

$$\rho^2 = \sigma^2 + \pi^2 = v^2. \tag{1.21}$$

The variable $\rho$ or $\sigma$, and therefore the physical scalar $H$, is eliminated from the effective theory as an independent scalar degree of freedom:

$$\sigma = (v^2 - \pi^2)^{1/2}. \tag{1.22}$$

Note that the constraint (1.21) is invariant under $SO(4)$ or $SU(2) \times SU(2)$. When the condition (1.22) is imposed, the linear transformations

$$\delta\pi_i = \epsilon_{ijk}\alpha_j\pi_k + \beta_i\sigma,$$

$$\delta\sigma = -\beta_i\pi_i, \tag{1.23}$$

where $\alpha_i$ and $\beta_i, i = 1,2,3$ are the parameters of, respectively, a "vector" and an "axial" $SU(2)$, are replaced by the nonlinear transformations

$$\delta\pi_i = \epsilon_{ijk}\alpha_j\pi_k + \beta_i(\pi^2 - v^2)^{1/2}. \quad (1.24)$$

The Lagrangian (1.10) takes the form

$$\mathcal{L}_H \to \frac{1}{2}\partial_\mu\pi^i\partial^\mu\pi^j g_{ij} \quad (1.25)$$

where

$$g_{ij} = \delta_{ij} + \frac{\pi_i\pi_j}{v^2 - \pi^2} \quad (1.26)$$

is the scalar metric. One can check that (1.25) is explicitly invariant under (1.24).

The Lagrangian (1.25) defines an effective nonrenormalizable theory, that, according to the equivalence theorem stated above, describes [12,7,13] the strong self-couplings of longitudinally polarized $W$'s and $Z$'s in the c.m. energy region $m_W << s << m_H^2$ in the large $m_H$ limit of the standard model. Although the theory is strongly coupled, invariance under chiral $SU(2)$, Eq. (1.24) assures[14] that the low energy limit of $S$-matrix elements for $\pi - \pi$ (and hence $W_L, Z_L$) scattering are given precisely by the Born, or tree, approximation to the Lagrangian (1.25):

$$S = S_{\rm Born}(1 + O(s/16\pi^2v^2)). \quad (1.27)$$

This is because (1.25) is the only form invariant under (1.24) that is at most quadratic in momenta (i.e., in derivatives).

1.3. The one-loop scalar action.

In this section I will outline a loop-expansion procedure for the effective action that explicitly preserves the invariances of the tree action. I start by recalling elements of functional integration, background field methods and the derivative expansion.

Consider first a free scalar field theory, with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi^i\partial^\mu\varphi^i - m^2\varphi^i\varphi^i), \ \ i = 1, \ldots, N. \quad (1.28)$$

The effective quantum action is

$$S_{\rm eff} = i \ln \int d^N\varphi e^{-S[\varphi]} \quad (1.29)$$

where the tree action as a functional of $\varphi$ is given by

$$S[\varphi] = \int d^4x \mathcal{L}(\varphi) = -\frac{1}{2}\int d^4x\varphi^i(x)\Delta^{-1}(x,y)\varphi^i(y)d^4y. \quad (1.30)$$

The inverse propagator

$$\Delta^{-1}(x,y) = (\partial_x^2 + m^2)\delta(x-y) \quad (1.31)$$

can be considered as an infinite dimensional matrix including the space time position $x$ as a matrix index. Then the integration (1.29) can readily be performed using the gaussian integral

$$\int d^N\chi e^{-\frac{1}{2}\chi_i M_{ij}\chi_j} = \det{}^{-1/2} M \quad (1.32)$$

which gives for (1.29)

$$S_{eff} = i \ln \det {}^{1/2}\Delta. \tag{1.33}$$

For a (renormalizable) interacting field theory, with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi^i\partial^\mu\varphi^i - V(\varphi) \tag{1.34}$$

the effective action is, in practice, evaluated as an expansion in perturbation theory. In the background field method one expands the functional $S[\varphi]$ around a classical background field configuration $\varphi_0$. Setting $\varphi = \varphi_0 + \hat{\varphi}$:

$$S[\varphi] = S[\varphi_0] + \frac{\delta S}{\delta\varphi^i}\bigg|_{\varphi_0}\hat{\varphi}^i + \frac{\delta^2 S}{\delta\varphi^i\delta\varphi^j}\bigg|_{\varphi_0}\hat{\varphi}^i\hat{\varphi}^j + \cdots. \tag{1.35}$$

The first term in (1.35) is the effective tree action expressed in terms of $\varphi_0$. The second term vanishes by virtue of the classical equations of motion in the presence of a background field $\varphi_0$. More precisely, one adds a source term $J_i(\varphi_0)\hat{\varphi}^i$ to the Lagrangian (1.34) which assures that the equations of motion are satisfied for $\varphi = \varphi_0$. The third term in (1.35) determines the one-loop correction to the effective action. Inserting (1.35) into (1.29) gives[16]

$$S_{eff} = S[\varphi_0] + i\ln\int d^N\hat{\varphi}\exp(-\frac{i}{2}\int dx\hat{\varphi}^i(\Delta^{-1})_{ij}\hat{\varphi}^j) + \cdots$$

$$= S[\varphi_0] + i\ln\det {}^{1/2}\Delta + \cdots = S[\varphi_0] + \frac{1}{2}iTr\ln\Delta + \cdots. \tag{1.36}$$

Here $\Delta$ is the propagator in the presence of the background field $\varphi_0$; defining the (background field dependent) "mass matrix"

$$U_{ij}(x) = U_{ij}[\varphi_0(x)] = \frac{\partial^2 V}{\partial\varphi^i\partial\varphi^j}\bigg|_{\varphi_0},$$

we obtain

$$\Delta^{-1}_{ij}(x,y) = (\partial^2_x + U_{ij}(x))\delta(x-y) = (\partial^2_x + U_{ij}(x))\int\frac{d^4p}{(2\pi)^4}e^{ip(x-y)}$$

$$= \int\frac{d^4p}{(2\pi)^4}e^{-ipx}(-p^2 + U_{ij}(-i\partial/\partial p))e^{ipy}. \tag{1.37}$$

By thus expressing the inverse propagator in terms of its Fourier transform, the x-integrations implicit in (1.36) become trivial, and as the p-dependence reduces to products of $\delta$-functions one obtains[16]

$$Tr\ln\Delta^{-1} = \int dx\int\frac{d^4p}{(2\pi)^4}Tr\ln(-p^2 + U(x - i\partial/\partial p)). \tag{1.38}$$

The remaining p-integration can be performed after a Wick rotation and a suitably defined expansion of the logarithm in (1.38) with

$$U(x - i\partial/\partial p) = U(x) - i\partial_\mu U(x)\partial/\partial p_\mu + \cdots \tag{1.39}$$

which gives the one-loop effective action as a series in increasing orders of derivatives[16].

In the case of a scalar theory with derivative couplings, the above formalism must be generalized to provide an expansion that, at each loop order, is manifestly invariant under field redefinitions. Consider a general $\sigma$-model with the Lagrangian

$$\mathcal{L} = \frac{1}{2} g_{ij}(\varphi)\partial_\mu\varphi^i\partial_\mu\varphi^j - V(\varphi). \tag{1.40}$$

Under a change of field variables:

$$\varphi^i \to Z^a(\varphi), \ \partial_\mu Z^a = \frac{\partial Z^a}{\partial\varphi^i}\partial_\mu\varphi^i \tag{1.41}$$

the scalar metric is redefined according to

$$g_{ij}(\varphi) \to h_{ab}(Z) = \frac{\partial\varphi^i}{\partial Z^a}\frac{\partial\varphi^j}{\partial Z^b} g_{ij}. \tag{1.42}$$

The integration measure $d^N\varphi$ in the expression (1.29) for the effective action must now be replaced[17] by the invariant measure $d^N\varphi \det^{1/2} g(\varphi)$, and a covariant expansion is obtained[18-20] by replacing the functional derivatives $\delta/\delta\varphi^i$ in (1.35) by covariant functional derivatives $\hat{D}_i$:

$$S[\varphi] = S[\varphi_0] + \hat{D}_i S\Big|_{\varphi_0}\hat{\varphi}^i + \hat{D}_i\hat{D}_j S\Big|_{\varphi_0}\hat{\varphi}^i\hat{\varphi}^j + \cdots. \tag{1.43}$$

As previously, the second term on the right in (1.43) vanishes by the equations of motion (with appropriate covariant source terms), and the third term determines the one-loop contribution which is governed by the inverse scalar propagator[19] for the theory (1.40) in the presence of a background field configuration $\varphi_0$:

$$\Delta_{ij}^{-1}(x,y) = \hat{D}_i\hat{D}_j S\Big|_{\varphi_0} = \frac{\delta^2 S}{\delta\varphi^i\delta\varphi^j}\Big|_{\varphi_0} - \Gamma^k_{ij}(\varphi_0)\frac{\delta S}{\delta\varphi^k}\Big|_{\varphi_0}\delta(x-y) \tag{1.44}$$

where $\Gamma$ is the scalar connection determined in the usual way from the scalar metric $g$. Explicit evaluation of (1.44) gives[20]

$$\Delta_{ij}^{-1}(x,y) = -g_{ik}(\varphi_0)[d^2 + U + R]^k_j\delta(x-y), \tag{1.45}$$

with

$$U^j_i \equiv g^{jk}D_kD_iV(\varphi), \ \ R^j_i \equiv R^j_{\ell mi}\partial_\mu\varphi^\ell\partial^\mu\varphi^m, \tag{1.46}$$

where $D_j$ is the covariant scalar derivative, analogous to the covariant functional derivative in (1.44), $R^j_{\ell mi}$ is the scalar curvature tensor, and

$$(d_\mu)^j_i = \partial_\mu\delta^j_i + \Gamma^j_{ik}\partial_\mu\varphi^k \equiv [\partial_\mu + \gamma_\mu(\varphi)]^j_i \tag{1.47}$$

is a scalar field redefinition covariant four-derivative. Inserting the above results into the quantum action (1.29) and using (1.32) we obtain

$$\begin{aligned} S^{1-loop}_{eff} &= -\frac{i}{2}Tr\ln g^{-1}\Delta^{-1} = -\frac{i}{2}Tr\ln[d^2 + U + R] \\ &= -\frac{i}{2}\int d^4x\int\frac{d^4p}{(2\pi)^4}Tr\ln A(p, x - i\partial/\partial p) \end{aligned} \tag{1.48}$$

with

$$A(p, x - i\partial/\partial p) = (ip_\mu - \tilde{\gamma}_\mu)^2 + \tilde{U} + \tilde{R}, \tag{1.49}$$

where, for an arbitrary (matrix-valued) function $F(x) \equiv F(\varphi_0(x))$, I define the corresponding barred function by

$$\bar{F}(x) \equiv F(x - i\partial/\partial p) = e^{-i\partial\,\partial/\partial p}F(x)e^{i\partial\,\partial/\partial p}$$
$$= F(x) - i\partial_\mu F\partial/\partial p_\mu + \cdots . \tag{1.50}$$

The derivative expansion (1.50) is not term-by-term covariant under scalar field redefinitions. An explicitly covariant expansion is obtained[20] by noting that if we define

$$B = UAU^{-1},\ \ U = e^{-id\,\partial/\partial p}e^{i\partial\,\partial/\partial p}, \tag{1.51}$$

then

$$\int d^4x \int \frac{d^4p}{(2\pi)^4} Tr \ln A = \int d^4x \int \frac{d^4p}{(2\pi)^4} Tr \ln B, \tag{1.52}$$

where $d_\mu$ is defined in (1.47). The equality (1.52) holds because $\partial/\partial p$ acting on the far right of the integrands makes no contribution, nor, by integration by parts, does $\partial/\partial p$ acting on the far left. Under the transformation (1.51) the functions $\bar{F}$, Eq. (1.50), become:

$$\hat{F} \equiv U\bar{F}U^{-1} = e^{-id\partial/\partial p}F(x)e^{id\partial/\partial p} = F(x) - i[d, F]\partial/\partial p + \cdots , \tag{1.53}$$

which gives an expansion that is term-by-term covariant. Furthermore, we have[20]:

$$U(ip_\mu - \bar{\gamma}_\mu)U^{-1} = i(p_\mu + \tilde{G}_{\nu\mu}\partial/\partial p_\nu) \tag{1.54}$$

where the covariant operator $\tilde{G}_{\mu\nu}(\varphi, \partial/\partial p)$ is defined in terms of the scalar curvature and its covariant derivatives

$$G^j_{i\mu\nu} = [d_\mu, d_\nu]^j_i = \partial_\mu\varphi^l\partial_\nu\varphi^k R^j_{ilk}(\varphi),$$
$$\tilde{G}_{\mu\nu} = \frac{1}{2}G_{\mu\nu} - \frac{i}{3}[d_\rho, G_{\mu\nu}]\partial/\partial p_\rho + \cdots . \tag{1.55}$$

Assembling these results, we may write the one-loop effective action, Eq. (1.48), in the manifestly invariant form

$$S^{1-Loop}_{eff} = \frac{i}{2}\int d^4x \int \frac{d^4p}{(2\pi)^4} Tr \ln[-(p_\mu + \tilde{G}_{\mu\nu}\partial/\partial p_\nu)^2 + \hat{U} + \hat{R}]$$
$$= \Lambda^4 \times \text{constant} - \frac{1}{32\pi^2}\int d^4x Tr \Big\{\Lambda^2(U(x) + R(x))$$
$$-\frac{1}{2}\ln\Lambda^2\Big[(U(x) + R(x))^2 + \frac{1}{3}G_{\mu\nu}G^{\mu\nu}\Big] + \text{finite terms}, \tag{1.56}$$

where I have performed the momentum integration to display explicitly the divergent contributions. The leading quartic divergence is field independent and therefore irrelevant as long as we are not interested in gravitational interactions (i.e., in the value of the cosmological constant). For supergravity models that I will consider in Sect. 2 this term is exactly cancelled among bosonic and fermionic loop contributions. For the case of a constant background field, $\partial_\mu\varphi_0 = 0$, we have $R = G_{\mu\nu} = 0$, and the expression (1.56) reduces to the familiar Coleman-Weinberg result[21] for the one-loop effective potential:

$$S^{1-Loop}_{eff}|_{\partial_\mu\varphi_0=0} = -\frac{1}{32\pi^2}\int d^4x Tr(\Lambda^2 M^2(\varphi) + \frac{1}{2}M^4\ln(M^2/\Lambda^2) + \text{constant}), \tag{1.57}$$

with the identification $M^2(\varphi) \equiv U(\varphi)$ for the field-dependent mass matrix. With non constant background fields there is, in particular, an additional quadratically divergent term proportional to the scalar Ricci tensor:

$$TrR = -R_{ij}\partial_\mu\varphi^i\partial^\mu\varphi^j \tag{1.58}$$

which represents a one-loop correction to the scalar metric tensor $g_{ij}$.

1.4 The (gauged) nonlinear $\sigma$-model.

We can immediately apply the results of the preceding Section to the nonlinear $\sigma$-model defined by Eqs. (1.25) and (1.26). There is no potential so $U = 0$, and the scalar curvature is readily evaluated to give[20,22]

$$R^i_{jkl} = \frac{1}{v^2}(\delta^i_l g_{jk} - \delta^i_k g_{jl}), \tag{1.59}$$

and the Ricci tensor

$$R_{ij} = \frac{(1-N)}{v^2} g_{ij}, \tag{1.60}$$

where $N$ is the number of real scalars $\pi_i$, is in this case proportional to the metric tensor. This is because the expression (1.25) with metric tensor (1.26) is the only two-derivative form that is invariant under the $SU(2)\times SU(2)$ transformations (1.23). Combining the one-loop result (1.56) with the tree Lagrangian (1.25) we obtain for the one-loop corrected effective Lagrangian

$$\begin{aligned}\mathcal{L}_{eff} = & \frac{1}{2}g_{ij}\partial^\mu\pi^i\partial_\mu\pi^j\left(1 - \frac{(N-1)}{16\pi^2}\frac{\Lambda^2}{v^2}\right) \\ & + \frac{1}{64\pi^2}Tr\left(R^2 + \frac{1}{3}G_{\mu\nu}G^{\mu\nu}\right)\ln(\Lambda^2/?) + \cdots. \end{aligned}\tag{1.61}$$

The first term in (1.61) can be viewed as a renormalization of the pion fields and vev $v$:

$$\pi_R = Z\pi, \quad v_R = Zv, \quad Z^2 = 1 - \frac{(N-1)}{16\pi^2}\frac{\Lambda^2}{v^2} \tag{1.62}$$

The second, logarithmically divergent, term involves couplings that are not present at tree level. *The argument of the logarithm is necessarily dimensionless. However in* the massless $\sigma$-model, there is no scale parameter to scale the dimensionful cut-off – hence the question mark in (1.61). In this theory, successive terms in the derivative expansion are increasingly infrared divergent[23], although $S$-matrix elements are well defined. Thus to get a sensible answer we must resume the expansion. The correct four-point scattering amplitudes can be obtained simply by dimensional analysis[22]: since $R^2$ and $G^2$ are at least quartic in scalar fields, the only dimensionful quantity appearing in the formal expression (1.56) that can appear in the argument of the logarithm is the derivative operator. Thus the last term in (1.61) should be replaced by

$$\frac{1}{64\pi^2}Tr\left[R(\ln\frac{\Lambda^2}{\partial^2} + a)R + \frac{1}{3}G_{\mu\nu}(\ln\frac{\Lambda^2}{\partial^2} + a')G^{\mu\nu}\right] + O(\partial/\Lambda), \tag{1.63}$$

where $a$ and $a'$ are constants of order unity that cannot be reliably determined, as discussed in Sect. 1.1

Specializing to the case $N = 2$, which is appropriate for the large $m_H$ limit of the standard model(and for pion physics), we obtain, for example, for the $\pi^+\pi^-$ elastic scattering amplitude at one loop[22] (here I set $a = a' = 0$):

$$\mathcal{M}(\pi^+\pi^- \to \pi^+\pi^-) = -iu/v^2$$
$$+\frac{1}{64\pi^2 v^4}\{3s^2\ln(\Lambda^2/-s) + 3t^2\ln(\Lambda^2/-t) + 2u^2\ln(\Lambda^2/-u)$$
$$-\frac{1}{3}t^2\ln(\Lambda^2/-s) - \frac{1}{3}s^2\ln(\Lambda^2/-t) + \frac{1}{3}u^2[\ln(\Lambda^2/-s) + \ln(\Lambda^2/-t)]\}, \qquad (1.64)$$

a result which has been obtained previously[24], using different techniques, in the context of pion physics. In Eq.(1.64) $s, t$ and $u$ are the usual Mandelstam variables: $s > 0, u, t < 0$. The term proportional to $\ln(\Lambda^2/-s) \cong \ln(\Lambda^2/s) + i\pi$ contains the absorptive part due to on-shell rescattering.

In the large $m_H$ limit of the electroweak theory, Eq. (1.64) can be interpreted as the one-loop corrected amplitude for the elastic scattering of longitudinally polarized $W^+W^-$. The tree amplitude[6,7] is given by the first term in (1.64), which contributes to

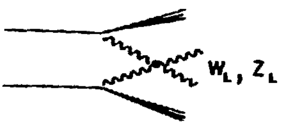


**Figure 3:** Vector boson fusion process for diboson production via strong $W_L, W_L$ rescattering in fermion collisions.

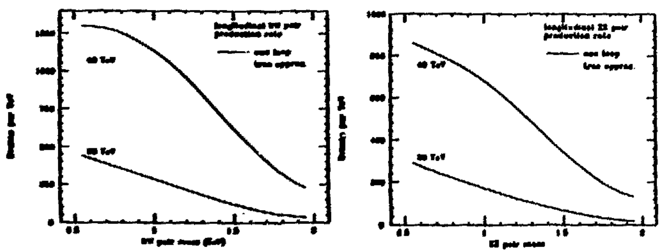

**Figure 4:** $W_L^+W_L^-$ and $Z_LZ_L$ pair production rates[22] in $pp$ collisions at $\sqrt{s} = 20$ and 40 TeV with a rapidity cut $|y| < 1.5$ and a cut-off $\Lambda = 3$ TeV. The amplitudes have been unitarized as described in the text.

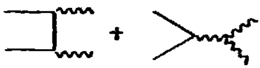

**Figure 5:** Diboson production via fermion-antifermion annihilation through renormalizable gauge couplings.

$s$ and $p$-wave scattering only. The one loop correction contains all partial waves (as well as a comparable amplitude[21] for elastic $Z_L^0$ scattering, which vanishes at tree level) and therefore represents a more realistic scattering model that incorporates the correct symmetry and analyticity properties, although it is not fully unitary. For $WW$ center of mass scattering energies $\sqrt{s_W} <$ 1 $TeV$, unitarity corrections are expected to be important only for the lowest ($s$) partial wave. Including a correction[22] for this, expected yields for $pp \to (Z_L Z_L$ or $W_L^+ W_L^-)$ + anything, via the fusion process of Fig. 3, are shown in Fig. 4 for a rapidity cut $|y| < 1.5$ and $pp$ c.m. energies of 20 and 40 $TeV$, where they can be compared with predictions[7] using (unitarized) tree amplitudes.

The one-loop corrections shown in Fig. 4 are surprisingly large, and one may question the usefulness of the one-loop approximation. The expansion parameters are effectively $s_{WW}/(4\pi v)^2$ and $(\Lambda/4\pi v)^2$, so the series converges for $s_{WW}, \Lambda^2 \lesssim (4\pi v)^2 \sim (3TeV)^2$. Thus if $\Lambda \lesssim 3TeV$, the results of Fig. 4 should be reliable in the energy range $m_W^2 << s_{WW} << \Lambda^2$, and will cease to be meaningful above the scale $\Lambda$ of "new physics" which could take the form of a Higgs scalar (or broad resonance in the $I = J = 0$ channel if $m_H \gtrsim TeV$) or a richer resonance spectrum. In the region $s_{WW} < \Lambda^2$, the experimental signature[13,7] for strong $W_L Z_L$ interactions is an enhancement of $WW, ZZ$ and $WZ$ production over what is expected from the scaling contribution from $q\bar{q}$ annihilation, Fig. 5. For $m_H \to \infty$, the tree contribution of Fig. 3 was found to exceed $q\bar{q}$ annihilation for $\sqrt{s_{WW}} \gtrsim (1/2 - 1)TeV$; the one loop corrections yield an even larger strong interaction contribution in the subresonance continuum region.

If we interpret the results of Fig. 4 and Eqs. (1.61)-(1.64) as applying to the large $m_H$ limit of the standard model, the underlying theory is renormalizable. We can compare these results with those obtained by calculating in the finite $m_H$, renormalizable theory, and then taking the large $m_H$ limit. For this purpose, we start with the linear $\sigma$-model of Eq. (1.10), in which case we have

$$R = G_{\mu\nu} = 0, \quad U_{ij} = \frac{\partial^2 V}{\partial\varphi_i \partial\varphi_j} \neq 0, \tag{1.65}$$

where I identify $(\varphi_0, \varphi_1, \cdots, \varphi_N) \equiv (\sigma, \pi_1, \cdots, \pi_N)$. The expansion (1.56) now gives

$$\mathcal{L}_{1-Loop} = \frac{-1}{32\pi^2}\left(\Lambda^2 TrU - \frac{1}{2}TrU^2 \ln \Lambda^2 + \text{finite terms}\right), \tag{1.66}$$

which, in particular contains no divergent derivative terms.

Now consider the limit $\lambda \to \infty$. It is convenient to introduce the variables $\rho$ and $\theta_i$:

$$\sigma = \rho\cos(\theta/v), \quad \pi_i = \rho\hat{\theta}_i \sin(\theta/v), \quad \theta = \left(\sum_{i=1}^{N} \theta_i^2\right)^{1/2}. \tag{1.67}$$

The potential is independent of the Goldstone modes $\theta_i$; exciting these modes with zero four-momentum costs no energy, even in the limit $\lambda \to \infty$. However, for $\rho \neq v$ the potential energy is infinite. As discussed in Sect. 1.2, $\rho$ remains fixed at its ground state value: $\rho^2 = \sigma^2 + \pi^2 = v^2$. In other words, to evaluate the effective action (1.29) we may introduce source terms for the $\theta_i$ but not for $\rho$. Imposing the classical equation of motion for $\rho$:

$$0 \equiv \frac{\delta S}{\delta\rho} = \left(\frac{\partial\sigma}{\partial\rho}\frac{\delta S}{\delta\sigma} + \frac{\partial\pi^i}{\partial\rho}\frac{\delta S}{\delta\pi^i}\right)\Bigg|_{\theta} \tag{1.68}$$

we can eliminate the background field $\sigma = \sigma(\pi_i, \partial_\mu \pi_i)$ in terms of the fields $\pi_i$ and their derivatives. The integral in (1.56) (or (1.37)) is most easily performed by first diagonalizing the "mass matrix" $U(\pi, \partial\pi)$. There is one eigenvalue

$$\text{"}m_\sigma^{2}\text{"} = \lambda(3\rho^2 - v^2) \underset{\lambda\to\infty}{\longrightarrow} \infty \tag{1.69}$$

that grows with $\lambda$ and decouples (up to a field-independent contribution) for $m_\sigma^2 >> \Lambda^2$, and $N$ eigenvalues

$$\text{"}m_{\pi_i}^2\text{"} = \lambda(\rho^2 - v^2) \underset{\lambda\to\infty}{\longrightarrow} \text{finite} \tag{1.70}$$

that remain finite in the limit. Since we are working with a renormalizable theory we can interpret $\Lambda$ as the renormalization scale. The effective theory obtained for a particular choice of $\Lambda$ is then a good approximation for energy scales in the neighborhood of $\Lambda$, and only light eigenmodes, $|\text{"}m\text{"}| \lesssim \Lambda$, contribute to the loop integrals for the effective theory. We expect the effective nonrenormalizable $\sigma$-model to be valid at scales much smaller than the Higgs mass $m_H \simeq \text{"}m_\sigma(\pi, \partial\pi)\text{"}$, i.e., for

$$\text{"}m_\pi^{2}\text{"} << \Lambda^2 << \text{"}m_\sigma^{2}\text{"}. \tag{1.71}$$

Indeed, when (1.66) (or more precisely (1.37)) is evaluated by taking the limit $m_\sigma \to \infty$ before the limit $\Lambda \to \infty$, the previous result, Eq. (1.61), is exactly reproduced[20,21].

The large $m_H$ limit of the standard electroweak theory is, in fact, a gauged nonlinear $\sigma$-model. The generalization of the above results to include background gauge fields $A_\mu$ is easily realized by replacing ordinary space-time derivatives by gauge-covariant derivatives:

$$\begin{aligned} \partial_\mu &\to D_\mu = \partial_\mu + iA_\mu \\ d_\mu = \partial_\mu + \gamma_\mu &\to d_\mu = D_\mu + \gamma_\mu. \end{aligned} \tag{1.72}$$

Then the expression (1.55) for $G_{\mu\nu}$ is modified to include a term proportional to the gauge field strength

$$G_{\mu\nu} = [d_\mu, d_\nu] = iF_{\mu\nu} + \cdots \tag{1.73}$$

and the logarithmically divergent contribution in (1.56) proportional to $G^2$ includes a term

$$G_{\mu\nu}G^{\mu\nu} = e^2 F_{\mu\nu}F^{\mu\nu} \tag{1.74}$$

that contributes to the one-loop $\beta$-function[20].

To fully determine the one-loop action, however, we must also include internal vector boson loops. This is complicated by the fact that when the tree action is expanded, as in (1.36) or (1.43), up to terms bilinear in the quantum fields (or functional integration variables) there are in general vector-scalar mixing terms:

$$\begin{aligned} S \ni \int d^4x |D_\mu\varphi|^2 &\ni \int d^4x\, i A_a^\mu T_{ij}^a \varphi^i \overleftrightarrow{\partial}_\mu \varphi_j^* \\ \ni \int d^4x \varphi_0 \hat{A}_\mu \partial^\mu \hat{\varphi}^* + h.c. &= -\int d^4x \hat{\varphi}\partial^\mu(\hat{A}_\mu \varphi_0) + h.c. \end{aligned} \tag{1.75}$$

To evaluate the effective potential[21] with $\varphi_0$=constant, $A_\mu = 0$ one usually works in the Landau gauge, $\partial_\mu \hat{A}^\mu = 0$, so that the last term in (1.75) vanishes identically and there is no vector-scalar coupling. When nonconstant scalar and vector background fields are present the situation is more complicated and one must find the gauge condition most appropriate for the specific calculation. The case relevant to the large $m_H$ standard model, namely the globally $SU(2) \times SU(2)$ symmetric nonlinear $\sigma$-model embedded in an $SU(2)_L \times SU(1)$ gauge group, turns out to be particularly complicated, but has been solved[28]. The divergent contributions to the effective scalar and gauge boson action have been determined, giving an expression of the form[28]

$$\mathcal{L}_{eff}^{1-loop} = -\frac{\Lambda^2}{16\pi^2}|D_\mu\varphi|^2 + \frac{\ln\Lambda^2}{16\pi^2}\{aF_{\mu\nu}F^{\mu\nu} + b|D_\mu\varphi|^2$$
$$+c(\varphi^\dagger D_\mu\varphi)^2 + \cdots\} + \text{finite terms.} \tag{1.76}$$

The first three terms in Eq. (1.76) can be interpreted as renormalization of fields and/or parameters of the tree Lagrangian. In fact parts of these logarithmically divergent contributions remain divergent for finite $m_H$. In particular, the coefficient $a$ determines the $\beta$-function for scales intermediate between $m_W$ and $m_H$.

The dots in the coefficient of $\ln\Lambda^2$ represent terms at least quartic in the gauge and scalar fields. According to the equivalence theorem of Sect. 1.2 we can calculate $S$-matrix elements by interpreting $A_\mu$ as a field operator for transversely polarized vector bosons and the $\pi_i$ in the expression (1.9) for $\varphi$ as field operators for longitudinal bosons. An examination of the exact expression [28,22] for these terms shows that there is a factor of the weak gauge coupling constant $g$ for each external transverse boson, and that the vertex functions with no external $A_\mu$ are precisely those obtained in the ungauged model.

In other words, the only divergent correction from gauge loops to the effective scalar action of Eq. (1.61) comes from the fourth term in Eq.(1.76), which has been identified[27] as the only two-derivative term that is $SU(2)_L \times U(1)$ gauge invariant but breaks global $SU(2) \times SU(2)$. It also contains a correction to the parameter

$$\rho \equiv m_W^2/m_Z^2\cos^2\theta_w. \tag{1.77}$$

In the unitary gauge:

$$(\varphi^\dagger D_\mu\varphi)^2\,|_{\pi=0} = -\frac{g^2v^2}{16\cos\theta_w}Z_\mu Z^\mu, \tag{1.78}$$

which contributes a shift in the $Z$-mass but not the $W$-mass Using the explicit value found[28] for $c$ in Eq. (1.75), one gets for the correction to the $\rho$-parameter (1.77):

$$\rho - 1 = \frac{-3g^2}{64\pi^2}\tan^2\theta_w \ln\left(\frac{\Lambda^2}{m_W^2}\right) + \text{finite} \tag{1.79}$$

which is well within experimental limits: $|\rho - 1| < 0.004$ if we take $\Lambda < 3TeV$ as discussed above. Conversely, experimental limits on $|\rho - 1|$ assure[27] that this term cannot contribute significantly to the $W_L, Z_L$ scattering amplitudes.

If we now set $\Lambda^2 = m_H^2$ in Eq. (1.79) the result is precisely that found[28] by taking the large $m_H$ limit of the one-loop corrected $\rho$-parameter as calculated in the renormalizable (finite $m_H$) standard model. Similarly, the logarithmically divergent four point functions (i.e., dots) in Eq. (1.76) agree[28] with previous results[29] found for those contributions that grow with $\ln m_H$ as calculated diagrammatically in the standard model.

We have thus established that one-loop effects calculated in the effective non-renormalizable theory defined by the $m_H \to \infty$ limit of the standard model agree with the large $m_H$ limit of one loop calculations evaluated using the renormalizable theory. This result lends a degree of credibility to the loop expansion of the effective nonrenormalizable theory.

On the other hand, the results shown in Fig. 4 are of much more general validity than the standard model. If the scalar sector possesses a chiral $SU(2) \times SU(2)$ symmetry, as mentioned in Sect. 1.2, the leading behavior of low energy $S$-matrix elements are necessarily those determined by the effective tree Lagrangian of Eqs. (1.25) and (1.26). There is only one possible gauge invariant, chiral symmetry breaking correction (Eq. (1.78)) to this low energy behavior and it is constrained to be small by observation: $\rho \simeq 1$. The effective tree Lagrangian (1.75) is therefore universal[27] up to corrections of order $|\rho - 1|$, and so, therefore, is the divergent part of the effective one-loop Lagrangian.

If in Eq. (1.26) we replace $v$ by $f_\pi$, the decay constant for $\pi \to \ell\bar{\nu}_\ell$, then Eq. (1.25) is the effective Lagrangian for pion physics, valid at energies $s_{\pi\pi} \lesssim m_\rho^2$ i.e., the resonance region in pion scattering. In this case the underlying renormalizable theory is (approximately) massless QCD, with Lagrangian

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_F} \bar{\psi}_i \gamma \cdot D\psi_i + G^a_{\mu\nu} G^{\mu\nu}_a \tag{1.80}$$

where $N_F$ is the number of quark flavors, $G^a_{\mu\nu}$ is a gluon field strength tensor and the covariant derivative is $D_\mu = \partial_\mu + ig_s \vec{\lambda^c} \cdot \vec{A}/2$, with $\lambda^c$ a $3\times3$ matrix operating on color indices. The Lagrangian (1.80) is invariant under global flavor $SU(N_F)_L \times SU(N_F)_R$ transformations on quarks:

$$\psi_{L,R} \to e^{i\alpha_{L,R}\lambda^F} \psi_{L,R}. \tag{1.81}$$

where $\lambda^F$ is an $N_F \times N_F$ matrix acting on flavor indices. Empirically, the first generation of quarks is very light, $m_u, m_d \simeq 0$, so chiral symmetry is a good approximation for $N_F = 2$. Experimental data tells us further that the vacuum is not chiral $SU(2)$ invariant. We attribute this observation to spontaneous symmetry breaking; the vacuum energy is lowest for $< \bar{\psi}\psi > \neq 0$. The quark condensate $< \bar{\psi}\psi >$ is not chiral invariant; its presence breaks chiral $SU(2)_L \times SU(2)_R$ to ordinary flavor $SU(2)$, i.e., the subgroup of transformations (1.81) with $\alpha_L = \alpha_R$. Spontaneous breakdown implies the existence of massless Goldstone bosons, which are assumed to be the (almost) massless pions. Chiral $SU(2)$ dictates that their low energy $S$-matrix elements be determined by the chiral invariant Lagrangian (1.25), (1.26). Loop corrections[24] then generate the one-loop effective contribution of Eqs. (1.61)-(1.64), where the effective expansion parameters are now $s_{\pi\pi}/(4\pi f_\pi)^2$ and $(m_\rho/4\pi f_\pi)^2$.

Technicolor is a nonstandard scenario for the spontaneous breaking of the electroweak gauge symmetry based on the extrapolation of the observed nonperturbative phenomena in QCD from the scale $\Lambda_{QCD} \sim 100MeV$ where color couplings become strong, to the scale $v \sim 250GeV$ of electroweak symmetry breaking. One assumes a new gauged technicolor interaction among techniquarks $\psi_T$ and techni-gauge bosons $A_T$ that is asymptotically free and strong at a scale $\Lambda_{TC} \sim 250\ GeV$. From the observation that

$$< \bar{\psi}\psi > \sim \Lambda^3_{QCD} \sim f^3_\pi \sim (100\ MeV)^3 \tag{1.82}$$

one infers that

$$< \bar{\psi}_T\psi_T > \sim \Lambda^3_{TC} \sim v^3 \sim (250\ GeV)^3. \tag{1.83}$$

The massless Goldstone bosons are technipions, $\pi_T$, the analogues of pions. The techniquarks are assumed to carry $SU(2)_L \times U(1)$ quantum numbers such that the condensate (1.83) also breaks the electroweak gauge symmetry. Then the technipions couple to the weak gauge bosons via the effective gauge invariant coupling (1.19), so that the $W$ and $Z$ acquire masses and "eat" the technipions which become their longitudinally polarized components. The equivalence theorem of Sect. 1.2 holds by construction, and chiral flavor invariance of the technicolor Lagrangian implies that (1.25),(1.26) is the effective technipion tree Lagrangian. Thus the results of this section apply specifically to technicolor models, and the yields of Fig. 4 are correct at energies below the technirho resonance mass where, of course, cross sections will rise dramatically.

The phenomenon of fermion condensation in a strongly coupled nonabelian gauge theory has also been invoked as a possible mechanism for breaking supersymmetry,[30,31] as will be discussed in my second lecture and the lectures of John Ellis.[32]

1.5 Supergravity and the gauge hierarchy problem.

The gauge hierarchy problem can be simply stated by noting that scalar masses have quadratically divergent loop corrections in nonsupersymmetric renormalizable theories. In general, if the theory possesses elementary scalar fields $\varphi$, the one-loop corrections will include mass terms:

$$\mathcal{L}_{1-loop} \ni a\frac{\Lambda^2}{16\pi^2}\varphi^2. \tag{1.84}$$

Technically, the term (1.84) can be reabsorbed into a renormalization, but the appearance of scalar masses much smaller than the natural scales of the theory, such as the grand unification scale $m_{GUT}$ or the Planck scale $m_P$, becomes very artificial. Moreover if the ultimate theory - including gravity - underlying observed physics is a finite rather than a renormalizable one, all mass parameters must be calculable in terms of the fundamental length scale (e.g., $m_P{}^{-1}$) of the theory.

In a theory with unbroken supersymmetry (SUSY) $a = 0$ identically in Eq. (1.84) because there is an exact cancellation between bose and fermi loop contributions to the scalar mass. Since SUSY is necessarily broken, the cancellation cannot be complete, but in the context of broken SUSY one anticipates an effective cut-off $\Lambda \sim m_{SUSY}$, i.e., the scale that governs boson-fermion mass splittings.

The scalar sector of the standard model, Eq.(1.8), is weakly coupled if the coupling constant $\lambda$ is small, $\lambda/4\pi \lesssim 1$, implying for the physical Higgs mass, Eq. (1.18), $m_H \lesssim 1\ TeV$. There is in fact no experimental evidence that the Higgs sector is not strongly coupled. On the other hand one must ultimately explain the known scale of electroweak symmetry breaking, $v \simeq 1/4\ TeV$. It is unlikely that this scale is orders of magnitude less than the scale parameter of the effective low energy scalar Lagrangian, even in the strongly coupled limit. In other words, experimental observation requires an effective cut-off less than or the order of a TeV.

In addition to SUSY, scalar masses (as in technicolor models) can be protected by spontaneously broken global symmetries. If $\varphi$ is the Goldstone boson of an exact symmetry of the Lagrangian that is spontaneously broken, it is necessarily massless and again $a = 0$ in Eq. (1.84). If there is a small explicit breaking of the global symmetry, $\varphi$ can acquire a correspondingly small mass. Consider for example, the

QCD Lagrangian, Eq.(1.80), but with quark masses included:

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}(m_q = 0) - m_u\bar{u}u - m_d\bar{d}d \cdots . \qquad (1.85)$$

The nonvanishing mass terms $m_{u,d} \neq 0$ explicitly break $SU(2)_L \times SU(2)_R$. An empirically good formula for the pion mass is:

$$m_\pi^2 \simeq \frac{m_{u,d}}{f_\pi} m_\rho^2 \equiv a\Lambda^2. \qquad (1.86)$$

Here the pion mass is governed by two effects: the scale $\Lambda \simeq m_\rho$ where the effective pion theory (1.25) breaks down and the ratio $a = m_{u,d}/f_\pi$ of explicit to spontaneous symmetry breaking. (There is no factor $(4\pi)^{-2}$ in (1.86) because $m_{u,d} \neq 0$ is a tree level effect.)

Now consider the minimal coupling of $N$ real scalar fields to gravity, with the action

$$S_O = \int d^4x \sqrt{g}(g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^i - \frac{m_P^2}{2}R). \qquad (1.87)$$

Here $g$ is the space-time metric ($\sqrt{g} \equiv \det^{1/2} g$) and $R$ is the space-time curvature. Loop corrections to the action (1.87) will generate divergent contributions to the scalar self-energy, Fig. 6a. In the supersymmetrized gravity theory, or unbroken supergravity, the contributions of Fig. 6a will be exactly cancelled by the gravitino ($\tilde{G}$) exchange diagrams of Fig. 6b.

Figure 6: Contributions to scalar ($\varphi$) self energy from (a) graviton (G) and (b) gravitino ($\tilde{G}$) loops. In Fig. 6b $\chi_i$ is the fermionic superpartner of $\varphi_i$.

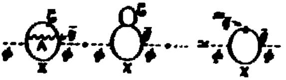

Figure 7: Two-loop contributions to scalar masses through combined gauge and gravitational interactions, which may be approximated as a one loop contribution with nonvanishing (at one loop) gaugino ($\tilde{g}$) mass.

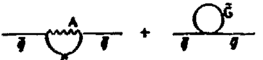

Figure 8: Gravitino-loop contributions to the gaugino mass; $A$ is a gauge boson.

When SUSY is broken the gravitino acquires a mass, $m_{\tilde{G}} \neq 0$, and the cancellation is no longer complete. Then one expects a (quadratically divergent) contribution to the scalar mass term:

$$m_\varphi^2 \sim \frac{m_{\tilde{G}}^2}{16\pi^2}\frac{\Lambda^2}{m_P^2} \tag{1.88}$$

where $\Lambda$ is the appropriate cut-off. If $\Lambda \sim m_P$, electroweak phenomenology requires $m_{\tilde{G}} \lesssim 10\ TeV$.

However the action (1.87), as well as its supersymmetric extension, is invariant[33] under global $SO(N)$ transformations among the $\varphi_i$. Thus to all orders the effective quantum action will depend on the scalar fields only through $SO(N)$ invariant quantities: $|\varphi|^2 = \sum_i \varphi_i^2$, $\sum_i \partial_\mu \varphi^i \partial^\mu \varphi^i$, etc. If the vacuum energy of the theory is lowest for a value $< |\varphi|^2 > \neq 0$, $SO(N)$ will break spontaneously to $SO(N-1)$, producing $N-1$ Goldstone bosons. Thus, only one of the $\varphi$'s will acquire a mass of the order of (1.88) while the $N-1$ others will remain massless to all orders.

In the real world, scalars have interactions other than gravitational ones. In particular there are gauge interactions that explicitly break the $SO(N)$ symmetry of the action (1.87), so one can expect *a priori* a (mild) suppression factor $a \sim \alpha$, where $\alpha$ is the gauge interaction fine structure constant. Suppose, however, that SUSY is broken, so $m_{\tilde{G}} \neq 0$, by the vev of a gauge singlet scalar. In the absence of gauge couplings $SO(N)$ is an exact symmetry of the Lagrangian, so the diagrams of Fig. 6 cannot generate scalar masses. On the other hand, if SUSY breaking is not communicated at tree level to the gauge sector, i.e., if the gaugino masses $(m_{\tilde{g}})_{tree} = 0$, gauge loop diagrams (see Fig. 11b below) vanish by supersymmetry. At the two-loop level, gauge interactions that know about $SO(N)$ breaking, and gravitational interactions, that know about SUSY breaking, can combine, as in Fig. 7, to yield nonvanishing gauge nonsinglet scalar masses that one might estimate[34] as:

$$m_\varphi^2 \simeq \alpha \frac{m_{\tilde{G}}^3}{(4\pi)^3}\frac{\Lambda^2}{m_P^3}, \tag{1.89}$$

requiring $m_{\tilde{G}} \lesssim 10^5\ GeV$ if $\Lambda \sim m_P$. One can estimate the two-loop contribution of Fig. 7 as a two-step process. First calculate the one-loop contribution, Fig. 8, to the gaugino mass (the blob in Fig. 7), and then use renormalization group equations to obtain the low energy value of the scalar masses, which should be of order

$$m_\varphi^2 \sim \frac{\alpha}{4\pi} m_{\tilde{g}}^2 \tag{1.90}$$

The two diagrams of Fig. 8 separately give contributions of the form:

$$m_{\tilde{g}} = \frac{a m_{\tilde{G}}}{(4\pi)^2}\Lambda^2/m_P^2 + \frac{1}{(4\pi)^2}(m_{\tilde{G}}^3/m_P^2)\left[b \ln(\Lambda^2/m_{\tilde{G}}^2) + c\right] \tag{1.91}$$

For $a \neq 0$, using (1.90), we would get the estimate (1.89). However, the divergent contributions from the two diagrams of Fig. 8, have been found[35] to cancel identically. Then if $c \neq 0$, using (1.91), we obtain instead of (1.89)

$$m_\varphi^2 \sim \frac{\alpha}{(4\pi)^5} m_{\tilde{G}}^6/m_P^4, \tag{1.92}$$

requiring only $m_{\tilde{G}} \lesssim 10^{-4} m_P \sim 10^{14}\ GeV$. Thus a large hierarchy for electroweak symmetry breaking could arise from a rather mild hierarchy for SUSY breaking relative to the Planck scale. In fact, subsequent calculations[46,47] showed that the two contributions of Fig. 8 to the gaugino mass cancel completely. In my second lecture I will discuss other sources of gaugino masses at one loop.

The above discussion is still unrealistic in that Yukawa couplings, which also break the $O(N)$ symmetry of the action (1.87) have still not been included - they are indeed necessary in the standard model for generating quark and lepton masses. Moreover, for a nonminimal gravitational coupling, i.e., for a nontrivial scalar metric, $g_{ij} \neq \delta_{ij}$, the gravitational action is not $SO(N)$ symmetric. It may however, as for the $\sigma$-model of Eqs. (1.25) and (1.26) possess a nonlinear symmetry that could play the same role. This is the case for a class of superstring inspired models, to be studied in Sect. 2, that possess a nonlinear noncompact global symmetry of the kinetic energy term.

A compact symmetry, such as $SO(N)$, leaves invariant the form $\sum_1^N \varphi_i^2$ under linear transformations, and, in particular, the canonical kinetic energy

$$\mathcal{L}_{K.E.} = \frac{1}{2}\sum_1^N \partial_\mu\varphi^i\partial^\mu\varphi^i \tag{1.93}$$

is $SO(N)$ invariant. A noncompact symmetry, such as $SO(m, N-m)$ leaves invariant the form $\sum_1^m \varphi_i^2 - \sum_{m+1}^N \varphi_i^2$ under linear transformations. The corresponding invariant kinetic energy term

$$\mathcal{L}_{K.E.} = \frac{1}{2}\left(\sum_1^m(\partial_\mu\varphi_i)^2 - \sum_{m+1}^N(\partial_\mu\varphi_i)^2\right) \tag{1.94}$$

is physically unacceptable as it contains "ghosts". Only nonlinear realizations of noncompact symmetries among scalar fields can lead to physically acceptable theories. For example the Lagrangian

$$\mathcal{L}_{K.E.} = \frac{\partial_\mu\varphi_i\partial^\mu\bar{\varphi}^i - (\varphi_i\partial^\mu\bar{\varphi}^i)(\bar{\varphi}^j\partial_\mu\varphi_j)}{(1-\varphi_i\bar{\varphi}^i)^2}, \quad i = 0, \cdots, N \tag{1.95}$$

where the $\varphi_i = (\bar{\varphi}^i)^\dagger$ are $N+1$ complex scalars, is invariant under nonlinear $SU(N+1,1)$ transformations. This can be seen most easily by writing (1.95) in the form

$$\mathcal{L}_{K.E.} = \partial_\mu\varphi_i\partial^\mu\bar{\varphi}^j G^i_j, \quad G^i_j = \frac{\partial^2}{\partial\varphi_i\partial\bar{\varphi}^j}G(\varphi,\bar{\varphi}), \tag{1.96}$$

which, as discussed by Ellis,[32] is the most general[48] form for the kinetic energy in $N+1$ supergravity theories. The real function $G(\varphi,\bar{\varphi})$ is the Kähler potential. For the Lagrangian (1.95) it is given by:

$$G = -\ln(1-\bar{\varphi}\varphi). \tag{1.97}$$

which is obviously invariant under compact, linear $SU(N+1)\times U(1)$ transformations. The remaining $2N+2$ transformations of $SU(N+1,1)$ are characterized by $N+1$ complex parameters $\alpha_i$ of the coset space $SU(N+1,1)/SU(N+1)\times U(1)$. Under the non linear transformations

$$\delta\varphi_i = \alpha_i - \varphi_i\bar{\alpha}^j\varphi_j \tag{1.98}$$

the Kähler potential is not invariant

$$\delta G = \bar{\alpha}\varphi + \bar{\varphi}\alpha. \tag{1.99}$$

However, since its variation is a sum of functions of $\varphi$ and of $\bar\varphi$, the Kähler metric $G^j_i$ and the hence kinetic energy terms are invariant.

In $N+1$ supergravity theories, as discussed by Ellis[37] the scalar potential (neglecting gauge-induced D-terms) is derived from the Kähler potential[38]

$$V(\varphi) = e^G(G_i(G^{-1})^i_j G^j - 3),$$

$$G_i = \frac{\partial G}{\partial \bar\varphi^i} = (G^i)^\dagger. \qquad (1.100)$$

For the Kähler potential (1.97), $V(\varphi)$ is invariant under linear $SU(N+1)$ transformations, since $G$ is, but it is not invariant under the nonlinear transformations (1.98). The Yukawa couplings, which are similarly derived[37,38] from the Kähler potential are also not invariant.

However the form of the kinetic energy term (1.95) does not uniquely determine the Kähler potential. To obtain an alternative Kähler potential we make the change of field variables.

$$T = \frac{1}{2}\frac{1+\varphi_0}{1-\varphi_0}, \quad C_i = \frac{\varphi_i}{1-\varphi_0}, \quad i = 1, \cdots N. \qquad (1.101)$$

Then (1.97) becomes

$$G = -\ln(T + \bar T - \bar C^i C_i) + f(\varphi) + \bar f(\bar\varphi). \qquad (1.102)$$

The first term in (1.102) appears in the Kähler potential for "no-scale" supergravity models[39] as well as some superstring-inspired models.[40,41] If instead of (1.97) we take the Kähler potential

$$G = -\ln(T + \bar T - C_i \bar C^i) \qquad (1.103)$$

we obtain the same kinetic energy, (1.96) which is $SU(N+1,1)$ invariant. The Kähler potential (1.103) is invariant, not under $SU(N+1,1)$, but under[42,43] a noncompact Heisenberg group $G_H$ of nonlinear global transformations.

$$C_i \to C_i + \alpha_i$$

$$T \to T + \bar\alpha C + \frac{1}{2}\bar\alpha\alpha + i\nu \qquad (1.104)$$

with $N$ complex parameters $\alpha_i$ and one real parameter $\nu$ of a compact axial $U(1)$ symmetry: $\delta\, Im\ T$ = constant. A supergravity theory defined by the Kähler potential (1.103) is, for vanishing gauge coupling constant, fully invariant under $G_H$ which can be shown[42] to imply $m_C = 0$ to all orders.

Neither (1.97) nor (1.103) defines a theory with realistic Yukawa couplings for the low energy theory. The class of superstring-inspired models that I will study in the following lecture have a Kähler potential of the form:[33]

$$G = -3\ln(T + \bar T - |C|^2) + \ln\ W(C) + \ln \bar W(\bar C) + \cdots, \qquad (1.105)$$

where the dots refer to functions of fields other than $T$ and $C_i$, and the super potential $W(C)$ generates the observed Yukawa couplings of the gauge nonsinglet sector $C$. Both $W(C)$ and the gauge couplings break invariance under (1.104). Nevertheless, as discussed in Sect. 2 below, $G_H$ invariance of the function (1.103) is sufficient[42] to protect scalar masses $m_C$ at one loop in the class of models defined by (1.105) that have a vanishing cosmological constant at tree level.

## 2. Superstring-Inspired Supergravity Models.

### 2.1 An effective tree potential.

In most of this lecture I will study a prototype model obtained by a simple compactification of 10-dimensional supergravity, with nonperturbative SUSY breaking effects incorporated.[34] At the end I will discuss the generalization of the results to a class of more realistic models. Ellis[33] has outlined the steps used in constructing the prototype model. Here I shall recall the relevant physical aspects and present the resulting potential.

Compactification from ten to four dimensions generally entails a number of scalar fields associated with the geometry of the compact manifold. In particular there is the dilaton field $\varphi_0$ related to scale transformations in 10-d supergravity, and the breathing mode $\sigma$ associated with fluctuations in the size of the compact manifold. The particular combinations

$$ReS = \varphi_0^{3/4}e^{3\sigma}, \quad ReT = \varphi_0^{3/4}e^{\sigma} + \frac{k}{2}|\varphi|^2, \tag{2.1}$$

where the $N$ complex fields $\varphi_i$ are gauge nonsinglets, are the scalar members of two chiral supermultiplets. In addition there are other gauge nonsinglet scalars associated with the detailed topology of the compact manifold that I will comment on later.

The possible relevance of these fields to phenomenology is that a) they couple only with gravitational strength to observed matter and thus provide the possibility of communicating weak SUSY breaking to the observed sector through quantum corrections, and b) they are associated with (classically) flat directions in the space of scalar field values. Specifically, if SUSY is unbroken, the effective tree potential in four dimensions is of the form

$$V = f(ReS, ReT)\tilde{V}(\varphi_i) \tag{2.2}$$

with $< \tilde{V}(\varphi_i) >= 0$, so the vevs of $ReS$ and $ReT$ remain undetermined at the classical level.

In order to make contact with observed physics, the vacuum degeneracy must be lifted and SUSY must be broken by nonperturbative quantum effects. Two sources of nonperturbative SUSY breaking have been proposed[30,31] in the context of the $E_8 \times E_8$ heterotic string.[44] With Calbai-Yau compactification,[32] for example, the gauge group in four dimensions is $E_6 \times E_8$, where $E_6$ is the gauge group of the observed sector and $E_8$ that of a hidden sector, coupled only gravitationally to observed matter. Both groups can be broken down further[32] at the compactification scale $\Lambda_{GUT}$ by loops of gauge flux trapped around topological singularities in the compact manifold. The surviving subgroup of $E_6$ must contain the observed $SU(3)_c \times SU(2)_L \times U(1)$ of the standard model for strong and electroweak interactions. The hidden gauge theory is assumed to be a pure supersymmetric Yang-Mills theory which is asymptotically free and therefore becomes strong at some scale $\Lambda_c$. This means that, as in QCD, Sect. 1.4, the gauginos of this strongly coupled sector may form a condensate:

$$< \bar{\lambda}\lambda > \propto h \neq 0 \tag{2.3}$$

which breaks supersymmetry[39] (as well as a chiral symmetry). In ten dimensional supergravity there is also a field strength $H_{LMN}(L, M, N = 0, \cdots, 9)$ that is an anti-symmetric, rank-three Lorentz tensor. This field may acquire a nonvanishing vacuum expectation value ($l, m, n = 5 \cdots, 9$):

$$< H_{lmn} > \propto c \neq 0 \tag{2.4a}$$

that satisfies a quantization condition[40]:

$$\int_S d\Sigma^{lmn} < H_{lmn} >= 2\pi n \tag{2.4b}$$

when integrated over a closed 3-surface $S$ of the six-dimensional compact manifold. The vev (2.4) also breaks supersymmetry. Either (2.3) or (2.4) alone would induce a positive cosmological constant. Combined they can contribute to the vacuum energy density in the form of a perfect square[31]

$$< V_{10} > \propto < (H + f(\varphi_0)\lambda\bar{\lambda})^2 > \tag{2.5}$$

which also involves the dilaton field $\varphi_0$. When one integrates over the compact 6-manifold to obtain the effective 4-d action the size of the compact manifold

$$\Lambda_{GUT} \sim m_P < e^{-2\sigma} >= m_P < (ReS\ ReT)^{-1/2} > \tag{2.6}$$

also appears, and the resulting potential depends on the scalar fields $S$ and $T$ in such a way that, for fixed values of the parameters $c$ and $h$, the degeneracy in $S$ is lifted. This is because it is the $S$-field that couples in four dimensions to the gauge bosons and gauginos. As a consequence its vev determines the unified gauge coupling constant:

$$< ReS >= (4\pi\alpha_{GUT})^{-1}. \tag{2.7}$$

Specifically, the full effective tree potential in this model takes the form:[40,31,28]

$$V_{tree} = U + \hat{V} + \mathcal{D} \tag{2.8}$$

with

$$U = (S + \bar{S})^{-1}(T + \bar{T} - k|\varphi|^2)^{-3}|W(\varphi) + c + h(1 + \omega)e^{-\omega/2}e^{-i\beta/2}|^2, \tag{2.9a}$$

$$\hat{V} = \frac{1}{3k}(S + \bar{S})^{-1}(T + \bar{T} - k|\varphi|^2)^{-2}\frac{\partial W}{\partial \varphi_i}\frac{\partial \bar{W}}{\partial \bar{\varphi}^i}, \tag{2.9b}$$

$$\mathcal{D} = \sum_a (3k\bar{\varphi}^i T^a{}_i^j \varphi_j)^2 (T + \bar{T} - k|\varphi|^2)^{-2}(S + \bar{S})^{-1}, \tag{2.9c}$$

where the matrices $T^a$ represent the generators of the observed gauge group on the chiral fields. In writing (2.9a) I have introduced the notation

$$\omega = \frac{3ReS}{2b_0}, \quad \beta = \frac{3ImS}{2b_0} \tag{2.10}$$

where $b_0$ governs the $\beta$-function of the strongly coupled hidden gauge sector. The superpotential $W(\varphi) = (\bar{W}(\bar{\varphi}))^\dagger$ is cubic in the gauge nonsinglet fields. $\hat{V}$ and $\mathcal{D}$ are, respectively the F-term and D-term that appear in globally supersymmetric theories, i.e., in the flat space limit $m_P \to \infty$, if supersymmetry is unbroken. Eqs. (2.9) are expressed in units of the reduced Planck mass:

$$1 \equiv m_P = (8\pi G_N)^{-1/2} \simeq 2 \times 10^{-18} GeV. \tag{2.11}$$

where $G_N$ is Newton's constant.

Each term in (2.8) is separately positive semi-definite. $\hat{V}$ and $\mathcal{D}$ are minimized for $\varphi_i = 0$ and therefore vanish at the ground state. If the SUSY breaking parameters $c$ and $h$ are absent, $W(\varphi) = 0$ forces $U = 0$ and the vevs of $S$ and $T$ are undetermined. When the supersymmetry breaking vevs of Eqs. (2.3) and (2.4) are turned on the vacuum energy vanishes for

$$\beta \equiv \beta_0 = 4\pi n, \qquad n \in Z, \tag{2.12a}$$

$$\omega \equiv \omega_0 : c = -h(1+\omega_0)e^{-\omega_0/2}. \tag{2.12b}$$

(The choice of sign in Eq. (2.12b) assures a CP-invariant $\theta$-vacuum, i.e., $F\tilde{F}$ does not contribute to the quantum action.) The vev of $T$ remains undetermined at tree level, as does the value of the gravitino mass:[20]

$$m_{\tilde{G}}^2 = < e^{\theta} > = < (S+\bar{S})^{-1}(T+\bar{T})^{-3}|c + h\, e^{-\omega/2}|^2 > . \tag{2.13}$$

At tree level there is therefore a four-fold vacuum degeneracy; in addition to $< ReT >$ and $< Im\, T >$, there is a two-fold degeneracy in the parameter space defined by $c, h$ and $\omega_0$. We shall now see to what extent this degeneracy is lifted at the one-loop level.

## 2.2 The Effective Theory at One Loop.

The effective one-loop potential is obtained by a covariant expansion of the quantum action with constant scalar background fields $z$, as in Eq. (1.43), but where now higher spin loops must be included. The result is the Coleman-Weinberg potential:[21]

$$\begin{aligned} V_{eff} &= V_{tree} + \frac{1}{2(4\pi)^2} Str \int dp^2 p^2 (p^2 + M^2(z)), \\ p^2 + M^2(z) &= Z^{-1}(z)\Delta^{-1}(p^2, z). \end{aligned} \tag{2.14}$$

$\Delta^{-1}(p^2, z)$ is the propagator in the presence of the background scalar fields $z$ and $Z(z)$ is a field dependent normalization matrix. For scalar loops $Z_{ij}^S(z) = g_{ij}(z)$, the scalar metric, and $M_S^2(z)$ is determined from the second covariant derivative of the potential, as discussed in Sec. 1.3. In a general supergravity model[20] the fermion and gauge boson kinetic energy terms are of noncanonical form. For example the fermion part of the Lagrangian is of the form

$$\mathcal{L}_F = \bar{\psi}^i [Z_{ij}^F(z)\gamma\cdot\partial + M_{Fij}^F(z)]\psi^j + O(\partial z) + O\left((\bar{\psi}\psi)^2\right) \equiv \bar{\psi}^i(\Delta_F^{-1})_{ij}\psi^j + \ldots \tag{2.15}$$

and the vector part is of the form

$$\begin{aligned} \mathcal{L}_V &= \frac{1}{4} f_{\alpha\beta}(z) F_{\mu\nu}^{\alpha} F^{\mu\nu\beta} + \frac{1}{2} A_{\mu}^{\alpha} \left(M_{V}^{V}(z)\right)^2_{\alpha\beta} A^{\beta\mu} + \cdots \\ &\equiv \frac{1}{2} A_{\mu}^{\alpha} (\Delta_V^{-1})_{\alpha\beta} A^{\mu\beta} + \text{ total deriv. } + \cdots . \end{aligned} \tag{2.16}$$

The corresponding mass matrices appearing in (2.11) are, respectively

$$\begin{aligned} M^F_{ij}(z) &= \left(Z_F^{-1/2}(z) M_0^F(z) Z_F^{-1/2}(z)\right)_{ij}, \\ \left(M^V(z)\right)^2_{\alpha\beta} &= \left(f^{-1/2}(z)\left(M_0^V(z)\right)^2 f^{-1/2}(z)\right)_{\alpha\beta}. \end{aligned} \tag{2.17}$$

I work in the Landau gauge, $\partial^\mu A_\mu = 0$, so that, as discussed in Sect. 1.4, the gauge bosons decouple from the scalar fields. Similarly, imposing the gauge condition $\gamma^\mu \psi_\mu = 0$ on the gravitino field $\psi_\mu$ assures that it decouples from fermions. The relevant part of the gravitino Lagrangian reduces simply to

$$\mathcal{L}_{\tilde{G}} = -\frac{1}{2}\bar{\psi}(\gamma\cdot\partial + m_{\tilde{G}}(z))\psi_\mu + \cdots. \tag{2.18}$$

With these gauge conditions the supertrace $F$ of a function of $M^2$ is defined by[46]

$$\begin{aligned} STr\ F(M^2) = 3Tr\ F(M_V^2) + Tr\ F(M_S^2) - 2Tr\ F(M_F^2) \\ - 4F(m_{\tilde{G}}^2) + 2F(4m_{\tilde{G}}^2), \end{aligned} \tag{2.19}$$

where the last term is the Fadeev-Popov "ghostino" contribution. The integral in Eq. (2.14) is divergent and must be regulated by a cut-off or subtraction parameter $\Lambda$. Neglecting terms of order $M^2/\Lambda^2$ we obtain:

$$V_{eff}(z) = V_{tree} + \frac{1}{32\pi^2}\left[\eta\Lambda^2 STr\ M^2 + \frac{1}{2}STr\ M^4 \ln(M^2/\rho\Lambda^2)\right], \tag{2.20}$$

where $\Lambda$ is the appropriate cut-off and $\eta$ and $\rho$ are prescription dependent parameters of order unity that reflect uncertainties in threshold factors and finite contributions as discussed in Sect. 1.1.

One can extract some of the qualitative features at one loop, that are independent of the precise shape of the effective potential, simply from dimensional analysis.[47] The only dimensionful quantities in (2.20) are the mass matrix $M$ and the cut-off $\Lambda$. Since the potential has dimension four the one-loop contribution is necessarily of the form

$$V_{1-loop} = STr\ M^4 f(M^2/\Lambda^2). \tag{2.21}$$

We wish to evaluate (2.21) in the neighborhood of the tree ground state, so we set $\varphi_i = 0$ and $\beta = \beta_0$. Then the elements of the squared mass matrix $M^2$ are all linear homogenous functions of the SUSY breaking parameters $c$ and $h$:

$$\begin{aligned} M^2(\varphi_i = 0) &= (ReT)^{-3}[a(\omega)ch + d(\omega)h^2] \\ &= h^2(ReT)^{-3}\nu(\omega) + O(\hbar), \end{aligned} \tag{2.22}$$

where in writing the last term in Eq. (2.22) I have used the tree level condition (2.12b) and $\hbar$ is the loop expansion parameter. The effective tree theory with potential (2.8), (2.9) is valid at scales below the scale of gaugino condensation, i.e., the scale where the hidden gauge interactions become strong, which is determined by renormalization group equations to be, using (2.6),

$$\begin{aligned} \Lambda_c^2 &= e^{-(4\pi b_0)^{-1}}\Lambda_{GUT}^2 = \left\langle e^{-\omega/3}\right\rangle \Lambda_{GUT}^2 \\ &= \left\langle e^{-4\pi-\omega/3}\right\rangle \end{aligned} \tag{2.23}$$

in Planck mass units. Defining the variables

$$\mu = h^2(ReT)^{-3}$$
$$\chi = \mu e^{4\omega} \tag{2.24}$$

we have $M^2 = \mu\nu(\omega)$, $M^2/\Lambda_c^2 = \chi\nu'(\omega)$, so identifying $\Lambda^2 = \Lambda_c^2$ in the supertrace (2.21) it takes the form

$$V_{1-loop} = \mu^2 f(\chi,\omega). \tag{2.25}$$

Near its ground state $\varphi_i = 0$ the tree potential is proportional to a perfect square:

$$V_{tree}(\varphi_i = 0) = (ReT)^{-3}\,|g(c,h,\omega)|^2 \tag{2.26}$$

with $g = 0$ when (2.12b) is imposed. Shifts of order $h$ from one-loop effects contribute at $O(h^2)$ to the tree level vacuum energy. Therefore to determine the one-loop vacuum configuration we need only retain the contribution (2.25). Since this is already $O(h)$, one loop corrections to the ground state condition (2.12b) will also contribute at $O(h^2)$.

At tree level there is a three-fold degeneracy in the parameter space defined by $(ReT)$, $\langle\omega\rangle$, $h$ and $c$. Thus, subject to the condition (2.12b), we must minimize the contribution (2.25) with respect to three independent variables in this space, that I take to be $\mu,\chi$ and $\omega$. The extrema of the one-loop corrected potential therefore occur for

$$\frac{\partial f}{\partial\chi} = \frac{\partial f}{\partial\omega} = 0 \tag{2.27a}$$

and

$$\mu^2 = 0 \text{ or } f = 0. \tag{2.27b}$$

Either of the conditions (2.27b) assures that the energy-density vanishes at all extrema of the potential. This implies that if the potential is not positive semi-definite everywhere it is unbounded from below. If the function $f(\chi,\omega)$ is positive semi-definite, there is always a global minimum at $\mu = 0$, for which supersymmetry is unbroken and all particles remain massless. If this is the only solution it means that one-loop corrections force the potentially SUSY breaking nonperturbative effects to vanish. As higher order perturbation corrections cannot break SUSY, this is not a physically acceptable solution.

If we impose the conditions (2.27a) with $\mu^2 \neq 0$, $f(\chi,\omega) = 0$ the function $f$ is overdetermined and a fine tuning of parameters other than the dynamical variables $\chi$ and $\omega$ is required for such a solution to exist. The theory contains no free parameters (such as coupling constants) other than the dynamical variables. This means that whether or not a nontrivial ($\mu^2 \neq 0$) solution exists depends on the detailed way in which the physics of the, presumably finite, underlying theory enters to damp the divergent integral (2.14). In numerical searches[47] for solutions to the minimization equations (2.27) we varied the uncertainty factors $\eta$ and $\rho$ using an approximation of the form (2.20). We considered a solution as acceptable if it occurs for plausible values of these parameters. If any such solution exists, and if the potential is bounded, it has vanishing vacuum energy and is infinitely degenerate, because the function $f$ is independent of the parameter $\mu$ that determines the scales of the theory. In other words, if one-loop corrections permit a vacuum with a finite, nonvanishing SUSY breaking gravitino mass the tree level degeneracy is lifted in all but one direction (aside from the axion, $ImT$, direction) in the space of dynamical variables. Thus the ratio $m_G/\Lambda_c$ is fixed, for example, but not the value of $m_G$. However the quantization condition (2.4b) implies that this degeneracy is discrete, and that all scales are fixed by the topology of the compact manifold.

I emphasize that, unlike the scalar field degrees of freedom, $h$ and $c$ are only parameters—not propagating fields—of the effective low energy theory valid at scales below $\Lambda_c$. There is a doubly infinite set of effective theories corresponding to possible choices for these parameters. Since they are, however, dynamical variables of the underlying theory they should relax or tunnel to those values that minimize the overall, fully quantum corrected vacuum energy. If there is any solution to (2.27) with finite $m_G$ there is one for any value of $\mu$, Eq. (2.24) and hence for any value of $c \propto h + O(\hbar)$. Once $c$ chooses one of its allowed values, all other vevs (except $\langle ImT\rangle$) are fixed.

I now assume that there exists a solution with finite gravitino mass. Soft supersymmetry breaking in the observable sector can be probed by expanding the one-loop effective theory around the ground state field configuration $z_0$. The $\varphi_i$ dependence of the effective potential can be obtained by writing the field dependent mass matrix as

$$M^2(z) = M^2(z_0) + \Delta \equiv M_0^2 + \Delta. \tag{2.28}$$

The supertrace of an arbitrary function $F(M^2)$ can then be expanded as

$$STr\ F(M^2) = STr\ F(M_0^2) + STr(\Delta F'(M_0^2)) + O(\Delta^2). \tag{2.29}$$

Since $\Delta = O(\varphi_i^2)$, the second term in (2.29) contains the quadratic and cubic $\varphi$-dependent terms that appear as soft SUSY breaking effects in the low energy, effective renormalizable theory.

In the most general supergravity models supersymmetry breaking, $m_G \neq 0$, at tree level induces both nonvanishing scalar masses,[38,46] proportional to $m_G$, and "A-terms" which are terms of order $m_G$ that are linear in the superpotential $W(\varphi)$. No such terms appear at tree level in the effective tree potential (2.8), (2.9) but they could appear at the one-loop level with coefficients suppressed by the loop factor $(4\pi)^{-2}$. An explicit evaluation of the mass matrix (2.28) gives, however, for the potential (2.21) when expanded as in (2.29), the following result.[47,48] If $V_{eff}$ is the one-loop corrected potential (2.14) and we define:

$$V(c,\ ReT,\omega) \equiv V_{eff}(\varphi_i = 0), \tag{2.30}$$

then the $\varphi$-dependence of $V_1$ is given by:

$$V_{eff}(z) = V(c + W(\varphi), ReT - \frac{k}{2}|\varphi|^2, \omega) + 0(\varphi^4) \tag{2.31}$$

which is precisely the form of the $\varphi$-dependence of $V_{tree}$ alone. In writing (2.30) I have not used the tree level condition (2.12b). If we now expand (2.31) up to terms cubic in the $\varphi_i$ we obtain

$$V_{eff} = V_{eff}(\varphi = 0) - \frac{k}{2}|\varphi|^2 \frac{\partial V_{eff}}{\partial ReT}$$
$$+ [W(\varphi) + \bar{W}(\bar{\varphi})]\frac{\partial V_{eff}}{\partial c}$$

$$+O(\varphi^4). \tag{2.32}$$

The ground state conditions $\partial V/\partial\, ReT = \partial V/\partial c = 0$ assure the vanishing of both the mass term and the "A-terms". Note that there is a quartic term in the expansion

$$V_{eff} \ni \frac{k^2}{8}|\varphi|^4 \frac{\partial^2 V_{eff}}{\partial ReT^2} \propto \frac{m^2_{ReT}}{m^2_{\tilde{g}}}|\varphi|^4 \tag{2.33}$$

that could lead to non-negligible SUSY breaking effects if $m^2_{ReT}$ is large. However it can be shown that this term disappears from the effective low energy theory for $\varphi_i$ when the heavy field $ReT$ is correctly integrated out.[42,49]

The vanishing of the scalar masses[50,51] can be traced[47] to the invariance of the form (1.103) under the Heisenberg group $G_H$ introduced in Sect. 1.5, as I will indicate more explicitly in Sect. 2.4. The vanishing of the A-terms[47] is less transparent; it occurs only when one minimizes the potential with respect to the parameters $c$ and $h$, as well as scalar vevs, and is therefore related to the vanishing of the cosmological constant. Large nonvanishing A-terms with vanishing scalar masses would be a phenomenological disaster, since all gauge nonsinglet scalars could acquire vevs, breaking, in particular, color and electric charge conservation.

Another possible source of soft supersymmetry breaking is gaugino masses. Since gauginos transform according to the adjoint representation of the gauge group, which is real, their masses, as for scalars, do not break the gauge symmetry. There are two sources for gaugino masses that are generated by radiative corrections. The first is from one-loop gaugino self-energy diagrams,[36,37] Figs. 8 and 9. As mentioned previously the diagrams of Fig. 8 cancel exactly, as do those of Fig. 9a.

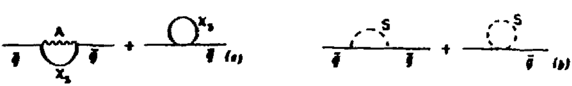


Figure 9: One-loop contributions to the gaugino mass from (b) the scalar field $S$ and (a) its chiral superpartner $\chi_S$.

The quadratically divergent contributions to Fig. 9b also cancel and the result gives a contribution[36,37] of order $m_{\tilde{g}} \sim m^3_{\tilde{G}} \ln(\Lambda^2/m^2_{\tilde{G}})$.

In addition there is a "tree-level" gaugino mass induced[37,52] by the shift at one loop in the tree level relation (2.12b). In the model[31] considered here, the tree-level gaugino mass is given by

$$(m_{\tilde{g}})_{tree} =< e^{\mathcal{G}/2}\frac{\partial \mathcal{G}}{\partial S}(S+\bar{S}) >=< U^{1/2} >= O(h) \tag{2.34}$$

where $\mathcal{G}$ is the Kähler potential and $U$ is defined in Eq. (2.9a) or (2.26). If, for example, $\omega_1 = \omega_0 + \delta\omega$ is the vev of $\omega$ as determined at one-loop, with $\omega_0$ given by (2.12), we get a contribution to the gaugino mass

$$\delta m_{\tilde{g}} = U^{1/2}(\omega_1) = \frac{1}{2}U^{-1/2}(\omega_0)\frac{\partial U}{\partial\omega}(\omega_0)\delta\omega. \qquad (2.35)$$

The shift $\delta\omega$ is determined by

$$\frac{\partial V}{\partial\omega}\Big|_{\omega_1} = 0 = \left(\frac{\partial U}{\partial\omega} + \frac{\partial V_{1-loop}}{\partial\omega}\right)\Big|_{\omega=\omega_1} = 0. \qquad (2.36)$$

Writing $U$ in the form, (2.26), we have

$$\frac{\partial U}{\partial\omega}\Big|_{\omega=\omega_1} = 2(ReT)^{-3}\left(\frac{\partial g}{\partial\omega}(\omega_0)\right)^2\delta\omega + O(\hbar^2),$$

$$\delta m_{\tilde{g}} \propto \delta\omega \propto -\frac{\partial V_{1-loop}}{\partial\omega}\Big|_{\omega=\omega_0}. \qquad (2.37)$$

When adding these two contributions care must taken to treat all divergent integrals in a consistent fashion. This can be done by evaluating the effective one-loop action in the presence of constant background gaugino as well as scalar fields. The term bilinear in gaugino fields, evaluated at the scalar ground state configuration, can then be identified with the gaugino mass term. The result found[*] in this way is that the two contributions to the gaugino masses cancel identically when one imposes the minimization conditions (2.27).

To show how such a cancellation can occur I will briefly outline the calculation. In the presence of both boson ($\varphi$) and fermion ($\psi$) background fields the inverse propagator can be written in the form[*]

$$\Delta^{-1}_{ij} = \hat{D}_i\hat{D}_j\, S_{eff}\big|_{\varphi,\psi} \equiv [Z(P+\delta)]_{ij} \qquad (2.38)$$

where $i,j$ refer to all quantum field degrees of freedom, $Z(\varphi)$ is the normalization matrix introduced in Eq. (2.14) and

$$P \equiv \mathcal{B}(\partial^2 + M_B^2(\varphi))\mathcal{B} + \mathcal{F}(i\gamma\cdot\partial + M_F(\varphi))\mathcal{F}, \qquad (2.39a)$$

$$\delta = \mathcal{B}\delta_{BF}\mathcal{F} + \mathcal{F}\delta_{FB}\mathcal{B} + \mathcal{B}\delta_{BB}\mathcal{B} + \mathcal{F}\delta_{FF}\mathcal{F} + O(\psi^3). \qquad (2.39b)$$

In Eqs. (2.39) $\mathcal{B}$ and $\mathcal{F}$ are projection operators on, respectively, the boson and fermion subspaces in the space of quantum fields (i.e., the functional integration variables $\hat{\psi},\hat{\varphi}$). Eq. (2.39a) determines the propagator for $\psi = 0$. The $\psi$-dependent part is expanded in Eq. (2.39b) where $\delta_{BF}$ and $\delta_{FB}$ are linear in $\psi$ and $\delta_{BB}$ and $\delta_{FF}$ are quadratic in $\psi$. The effective one-loop Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{1-loop} &= \frac{i}{2}\ln\int d\hat{\varphi}d\hat{\psi}\,\det{}^{1/2}Z(\varphi)e^{-\frac{1}{2}\int d^4x(\hat{\varphi}\,\hat{\psi})\Delta^{-1}\binom{\hat{\varphi}}{\hat{\psi}}} \\ &= -\frac{i}{2}STr\ln(P+\delta) \\ &= -\frac{i}{2}\Big\{STr\ \ln\ P + Str\,P^{-1}\delta - \frac{1}{2}STr\ P^{-1}\delta P^{-1}\delta + O(\psi^4)\Big\}. \qquad (2.40)\end{aligned}$$

The first term in brackets gives the effective one-loop bose Lagrangian, in particular the effective potential. It represents a sum of one-loop diagrams with any number of external bose fields. The other two terms correspond to diagrams with two external fermion lines. The second term has a closed scalar or fermion line as in the second diagram of Fig. 8, 9a and 9b, while the third term has one boson and one fermion internal line as in the first diagram of these figures. It is however easier to evaluate these terms by making a change of integration variables:

$$\tilde{\psi} \to \hat{\psi} = \hat{\psi} + (i\gamma \cdot \partial + M_F(\varphi))^{-1}\delta_{BF}. \tag{2.41}$$

In terms of the fields $\hat{\psi}$ and $\hat{\varphi}$ the propagator takes the diagonal form

$$Z^{-1}\Delta^{-1} = \mathcal{B}(\partial^2 + M_B^2(\varphi) + \Delta_B\bar{\psi}\psi)\mathcal{B}$$
$$+\mathcal{F}(\partial^2 + M_F^2(\varphi) + \Delta_F\bar{\psi}\psi)\mathcal{F} + O(\psi^4), \tag{2.42}$$

and, with the appropriate gauge conditions, the supertrace reduces to the form of Eq. (2.19) where the mass matrices $M$ now contain terms bilinear in $\psi$.

For the case of interest $\varphi \to z$, the set of scalar fields, and $\psi \to \lambda$, the background gaugino fields, and we obtain

$$-\mathcal{L}_{eff}^{1-loop} = STr\ F(\Lambda, M^2(z) + \Delta(z)\bar{\lambda}\lambda)$$

$$= STr\ F(\Lambda, M^2(z)) + \bar{\lambda}\lambda STr(\Delta(z)\frac{\partial}{\partial M^2}F(\Lambda, M^2(z)))$$
$$+O((\lambda\bar{\lambda})^2), \tag{2.43}$$

where I have expanded as in (2.29). The first term on the right in (2.43) is the scalar potential of Eq. (2.21): $F(\Lambda, M^2) \equiv M^2 f(M^2/\Lambda^2)$. The second term gives the one-loop gaugino self energy, Figs. 8 and 9. To compare this contribution with the one arising from the shift in the tree level relation (2.12b), I define

$$\delta c \equiv c + h(1 + \omega_0)e^{-\omega_0/2} = O(h). \tag{2.44}$$

Then by reasoning identical to that of Eqs. (2.34)-(2.37) we have

$$\delta m_{\frac{1}{2}} \propto \delta c \propto -\frac{\partial V_{1-loop}}{\partial c}. \tag{2.45}$$

It is straightforward to verify that when the minimization equations (2.27) are imposed (and the appropriate coefficients included) Eqs. (2.37) and (2.45) give the same result. Since $V_{1-loop}$ depends on $c$ only through the squared mass matrix $M^2$, we obtain

$$\delta m_{\frac{1}{2}} \propto -\frac{\partial V_{1-loop}}{\partial c} = STr\left[\frac{\partial M^2(\varphi)}{\partial c}\frac{\partial}{\partial M^2}F(\Lambda, M^2)\right]. \tag{2.46}$$

The right-hand side of (2.46) and the last term in (2.43) are supertraces over the same matrix valued function, $\partial F(\Lambda, M^2)/\partial M^2$, with different weight matrices, namely $\partial M^2/\partial c$ and $\Delta(z)$. These matrices can be calculated, and one finds that when the ground state conditions are imposed:

$$0 = STr\ F = \frac{\partial}{\partial\omega}STr\ F = \frac{\partial\Lambda}{\partial\omega}STr\frac{\partial F}{\partial\Lambda} + \frac{\partial M^2}{\partial\omega}\frac{\partial F}{\partial M^2}, \tag{2.47}$$

the two contributions cancel identically, independently of the functional dependence of $f(M^2/\Lambda^2) = M^{-2}F(\Lambda, M^2)$ on its argument.

2.3 The stability of the potential.

The results of the preceding section were obtained under the assumption that the effective one-loop potential is bounded from below. If the approximate form (2.20) is used, with $\eta, \rho \simeq 1$, the potential is indeed bounded[51] for $\omega_0 > \frac{1}{8}$ $(\alpha_{GUT} \lesssim 1)$ as a function of $ReT$ with c and h fixed, but it is negative at its minimum in this direction. In view of the results of Sect. 2.2, this implies that it slopes to $-\infty$ in some direction in the $(c, h)$ plane. Moreover, examination[52] of the $O(M^2/\Lambda^2)$ corrections to the approximation (2.20) shows that they destabilize the potential in the direction $ReT \to 0$, or $m_G \to \infty$. Explicitly, for $M^2 >> \Lambda^2$, the integral (2.14) becomes

$$V_{1-loop} \simeq \frac{1}{64\pi^2}\Lambda^4 STr \ln(M_H^2/\Lambda^2) \tag{2.48}$$

where the notation $M_H^2$ implies that the supertrace is over the subspace of massive modes: $m^2 >> \Lambda^2$. The stability of the potential therefore depends on whether there are more massive bosonic or fermionic states :

$$Sign(V)_{T\to 0} = Sign(B - F)_{m^2 \neq 0}. \tag{2.49}$$

For the theory corresponding to the tree potential (2.8),(2.9), one finds $(B - F)_{m^2\neq 0} = -4$, and the potential is unstable. However loop corrections calculated for this theory are not valid at field values for which $M^2 > \Lambda_c^2$, since large $M^2(z)$ in the loop propagators probes comparably large momenta. At scales larger than the condensate scale $\Lambda_c$ the gauge couplings are weak and there is no gaugino condensation. We expect[47] the effective theory relevant at scales between $\Lambda_c$ and the compactification scale $\Lambda_{GUT}$ to be approximately described by the potential (2.8) but with $h = 0$ in Eq. (2.9a). The mass spectrum of the corresponding effective supergravity theory, evaluated at the ground state $z = z_0$ of the tree potential with $h \neq 0$, satisfies[47]

$$(B - F)_{m^2\neq 0} = 2N - 2N_G - 3 \equiv 2\Delta + 1 \tag{2.50}$$

where $N$ is the number of chiral supermultiplets and $N_G$ the number of gauge multiplets, so the potential is bounded if

$$\Delta = N - N_G - 2 \geq 0. \tag{2.51}$$

Of course one-loop corrections calculated for the effective theory with $h = 0$ also cease to be valid for $M^2(z) > \Lambda_{GUT}^2$. However the condition (2.51), if satisfied, assures that an apparently stable ground state found using an approximation like (2.20) will not be simply an artifact of that approximation.

The results of the preceding section imply that the potential is unbounded in some direction of parameter space unless it is positive definite everywhere. Since $V(M^2(z)) = 0$ for $M^2(z) = 0$, this implies in particular that the slope at the origin of $M^2$ must be positive. The behavior at small $M^2$ is governed by the quadratically divergent term in (2.20), proportional to $STrM^2$.

For the supergravity theory defined by the potential (2.8), (2.9) (and by the gauge field normalization matrix, Eq. (2.16), $f_{\alpha\beta}(z) = \delta_{\alpha\beta}S$) one finds[51]

$$STrM^2 = 2\Delta U - 2(e^{\varphi} - U) + O(\varphi^4). \tag{2.52}$$

For $h \neq 0, U = \varphi_i = 0$ and $e^{\hat{\nu}} > 0$ at the tree ground state, so $STrM^2 < 0$, and the potential is unbounded in the direction $m_{\tilde{G}}^2 = e^{\nu} \to \infty$. For $h = 0$, $U \equiv e^{\nu}$, so the slope at the origin of $m_{\tilde{G}}$ for $\varphi_i = 0$ depends on the sign of $\Delta$, defined in Eq. (2.50).

If we split the loop integrals into two regions

$$a) \qquad |p|^2 \leq \Lambda_c^2, \; h \neq 0, \tag{2.53a}$$

$$b) \qquad \Lambda_c^2 \leq |p|^2 \leq \Lambda_{GUT}^2, \; h = 0, \tag{2.53b}$$

the effective one-loop potential takes the general form:

$$V_{1-loop} = STrV_a(M^2, \Lambda_c^2) + STrV_b(\tilde{M}^2, \Lambda_{GUT}^2, \Lambda_c^2),$$

$$V_a = M^4 F_a(M^2/\Lambda_c^2)$$

$$V_b = \tilde{M}^4 F_b(M^2/\Lambda_{GUT}^2, \Lambda_c^2/\Lambda_{GUT}^2), \tag{2.54}$$

where $M^2$ and $\tilde{M}^2$ are, respectively, the appropriate mass matrices for regions (a) and (b) of integration. If the quadratically divergent term in $V_{(b)}$ is positive and dominates that in $V_{(a)}$, the slope at the origin of $m_{\tilde{G}}^2$ will be positive and the potential may be positive semi-definite everywhere.[47,48] This requires in particular $\Delta > 0$, or since $\Delta$ as defined by Eq.(2.50) is an integer

$$\Delta \geq 1. \tag{2.55}$$

However, with the inclusion of one-loop corrections to condensate effects, to be discussed below, the interpretation of the effective parameter $\Delta$ that actually governs the slope at the origin may be modified, and it is not necessarily an integer.

The mass matrix relevant to region (b) is of the form

$$\tilde{M}^2(\varphi_i = 0) = c^2(ReT)^{-3}\tilde{\nu}(\omega) = h^2(ReT)^{-3}\tilde{\nu}'(\omega) + O(h) = \mu\tilde{\nu}'(\omega), \tag{2.56}$$

where I have used (2.12b), and since $\Lambda_c^2/\Lambda_{GUT}^2$ (see Eq. (2.23)) depends only on $\omega$, the modified one-loop potential (2.54) is still of the form (2.25). Then the reasoning leading to the conditions (2.27), and the conclusions of Sect. 2.2 regarding the cosmological constant, are still valid.

Using approximations of the form (2.20) for both terms in (2.54), the potential has been studied[47,48] numerically by varying its parameters. Solutions to the minimization equations were found for plausible values of the uncertainty factors $\eta_i$ and $\rho_i$, small values of $\Delta$ and values of $\omega_0$ in the range $2 \lesssim \omega_0 \lesssim 5$. This corresponds, via Eqs. (2.7) and (2.10), to $1/16 \lesssim \alpha_{GUT} \lesssim 1$ where I assume that

$$.06 \leq b_0 \leq 0.56, \tag{2.57}$$

i.e., that the hidden gauge group $G_{hid}$ satisfies $SU(3) \subset G_{hid} \subset E_8$. The potential for one such solution is shown in Fig. 10. As the vacuum is degenerate absolute

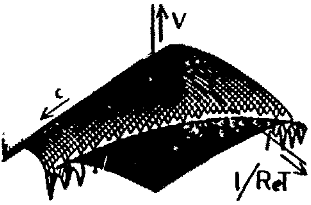


Figure 10: The one-loop effective potential in the $c-(\mathrm{Re}\,T)^{-1}$ plane for fixed values of the other dynamical variables in the case where a minimum exists for finite $m_{\tilde{G}}$.

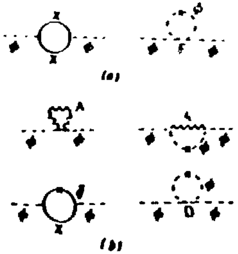


Figure 11: One loop contributions to the scalar ($\varphi$) self energy in a renormalizable SUSY theory which vanish when tree level masses (x) vanish.

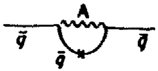


Figure 12: Gaugino mass renormalization for nonvanishing tree-level gaugino mass (x).

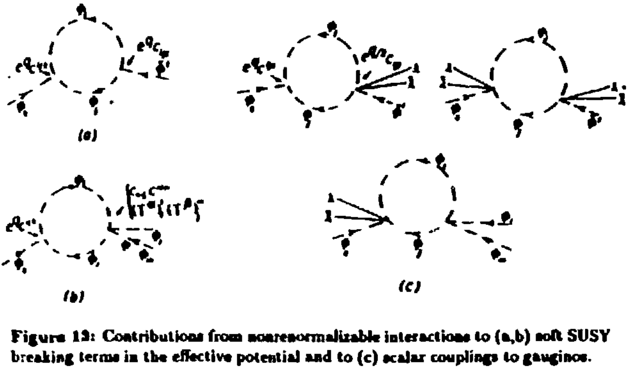


Figure 13: Contributions from nonrenormalizable interactions to (a,b) soft SUSY breaking terms in the effective potential and to (c) scalar couplings to gauginos.

mass scales are not determined, but their ratios are determined; one finds

$$m_{\tilde{G}}/m_P \sim 0.1\tilde{\eta}^{1/2}\Lambda_{GUT}/m_P \sim \tilde{\eta}^{3/4}/\sqrt{cb_0}, \qquad (2.58)$$

where $\tilde{\eta}$ is one of the threshold factors.

As before, we can expand the potential (2.54), or the corresponding effective Lagrangian, about the ground state field configuration to study soft supersymmetry breaking in the observable sector. One finds[40] that there are again no "A-terms", i.e., terms proportional to the superpotential $W(\varphi)$. However, if one simply integrates the expression (2.14) over $\Lambda_c^2 < |p|^2 < \Lambda_{GUT}^2$ with $M^2$ replaced by $\tilde{M}^2$ one finds (including a *threshold uncertainty factor* $\tilde{\rho}$) soft SUSY breaking *terms in the potential* that are proportional to the factor

$$a(\omega) = \ln(\tilde{\rho}\Lambda_c^2/\Lambda_{GUT}^2). \qquad (2.59)$$

Note that this factor does not grow with the cut-off scales for fixed $\omega$. It is as ill-determined as any of the finite (i.e., cut-off independent) terms. The shape of the potential for $\varphi_i = 0$ is in fact not very sensitive to its presence; setting $a(\omega) = 0$ has little influence[40] on the characteristics of the solutions to the minimization conditions.

However, we wish to ascertain the presence or absence of soft SUSY breaking independently of the details of the potential; therefore we should assume a priori that $a(\omega) \neq 0$. We then find two types of SUSY breaking terms arising from region (b) of loop integration. First, for $h = 0$ and $c \neq 0$ gauge nonsinglet scalars and gauginos have SUSY breaking tree-level masses proportional to the gravitino mass. These masses are renormalized at one loop through the standard diagrams, Figs. 11 and 12, of a renormalizable (softly broken) SUSY gauge theory. These terms simply represent a renormalization of the parameters that define the theory at scales $\mu > \Lambda_c$ above gaugino condensation, and cannot change qualitatively the features of the physics at scales $\mu < \Lambda_c$. The mass terms generated by the diagrams of Figs. 11 and 12 would in fact vanish if we first renormalized (at one loop) the effective theory for $\mu > \Lambda_c$ and then let $< \bar{\lambda}\lambda > \neq 0$ to determine the effective theory for $\mu < \Lambda_c$.

A second source of soft SUSY breaking terms in the effective one-loop scalar potential is from nonrenormalizable interactions. Expanding the term $e^{\mathcal{G}}$ in the tree potential for region (b);

$$V_{(b)tree} = e^{\mathcal{G}} + \hat{V} + \mathcal{D} \qquad (2.60)$$

yields the one loop contributions of Figs. 13a,b to terms that are quadratic (mass terms) and cubic (but not proportional to W $(\varphi)$) in the gauge nonsinglet scalar fields. However, the effective scalar one-loop Lagrangian, including background gaugino fields, also *contains* the $\bar{\lambda}\lambda$-*dependent terms* generated by the diagrams of Fig. 13c. When the diagrams of Figs. 13a,b and 13c are added the $\varphi_i^2$ and $\varphi_i^3$ terms in the effective one-loop Lagrangian, as expanded about the $h \neq 0$ tree vacuum, are proportional[40] to $< e^{\mathcal{G}/2} + \frac{1}{4}\bar{\lambda}\lambda >$. On the other hand, the nonderivative part of the tree Lagrangian valid at scales $\mu > \Lambda_c$ is (including only scalar and gaugino fields)

$$\mathcal{L}(z,\lambda) - \mathcal{L}_{K.E}(z,\lambda) = (e^{\mathcal{G}/2} + \frac{1}{4}\bar{\lambda}\lambda)^2 + \hat{V} + \mathcal{D}. \qquad (2.61)$$

The vanishing of the tree level vacuum energy for a nonvanishing gaugino condensate:

$$< e^{\mathcal{G}/2} + \frac{1}{4}\bar{\lambda}\lambda >= 0 \qquad (2.62)$$

should also imply the vanishing, at scales below $\Lambda_c$ where $< \lambda\lambda > \propto h \neq 0$, of the soft SUSY breaking terms generated by the diagrams of Fig. 13c.

One might then wonder whether the contribution of region (b) of loop integration is entirely cancelled by one-loop contributions to gaugino condensation effects, in which case the slope of the effective one-loop potential would be negative at the origin of $m_{\tilde{G}}$. This is almost certainly not the case. The effective tree potential of Eqs. (2.8) and (2.9) that defines the effective theory for $\mu < \Lambda_c$ can be obtained from the effective nonderivative Lagrangian of Eq. (2.61) by the replacement $< \bar{\lambda}\lambda > \rightarrow f(z)$. The effective scalar mass matrix, obtained as the second (covariant) scalar derivative *of the effective Lagrangian is not invariant under this replacement:*

$$\frac{\partial}{\partial z} < \bar{\lambda}\lambda > = 0, \quad \frac{\partial}{\partial z} f(z) \neq 0. \tag{2.63}$$

One could therefore conjecture that the net effect of region (b) loop contributions, after inclusion of loop corrections to condensate effects, is only to modify the contribution of scalar loops. Using this conjecture one finds[40] that the effective value of $\Delta \rightarrow \Delta_{eff}(\omega)$ that governs the slope of the potential near $m_{\tilde{G}} = 0$ is a (generally noninteger) function of $\omega$, independent of $N$ and $N_Q$. A positive semi-definite potential can occur for $\omega < 1.7$ ($\alpha_{GUT} > 0.4$), and the value of $\Delta_{eff}(\omega)$ turns out to be naturally of order unity, which is consistent with the results of the numerical analysis described above that require a value $\Delta \sim 1$ for the existence of a solution to the minimization equations. The functional form of $\Delta_{eff}(\omega)$, and hence the condition $\omega < 1.7$, depends on the precise functional form the potential, Eq. (2.9a), while the qualitative results of Sect. 2.2 are independent of this.

However, the above reasoning is not really correct since one cannot obtain the effective Lagrangian, incorporating the correct symmetry properties, that is appropriate for the description of physics scales $\mu < \Lambda_c$ by a simple and unique substitution $\bar{\lambda}\lambda \rightarrow f(z)$ in the Lagrangian valid at scales $\mu > \Lambda_c$. The correct procedure [31,54] is to first determine the effective superpotential appropriate for scales $\mu < \Lambda_c$; the effective Lagrangian is then determined by the standard prescription[30] for $N = 1$ supergravity.

Therefore, to correctly incorporate one-loop effects from physics at scales $\mu > \Lambda_c$, one should first calculate the effective one-loop Lagrangian, including corrections to gaugino couplings, relevant at these scales. For the effective theory with $h = 0$, all the quadratically divergent contributions that have been calculated thus far[51,20,40,55] have the property that they are proportional to terms that appear in the tree Lagrangian of that theory. This strongly suggests that these terms can be interpreted as field and Kähler potential renormalizations in such a way that the tree plus quadratically divergent one-loop effective Lagrangian can be cast in standard form.[30] One could then define a corrected effective "tree" Lagrangian valid at scales just below $\Lambda_c$ following the procedure of Affleck et al.,[54] to which, of course, the one -loop corrections of Sect. 2.2 should be added. On the other hand, logarithmically divergent corrections involve[20,56] terms of higher order in space-time and Kähler derivatives and in the Kähler and space-time curvatures. Interpreting these corrections in a similarly consistent fashion would first require a generalization of the standard $N = 1$ supergravity Lagrangian to higher derivative terms.

*As mentioned above*, the structure of the effective potential relevant to the determination of the vacuum energy is insensitive to the presence of logarithmically divergent terms, Eq. (2.59). In particular, a determination of the quadratic divergences is sufficient to resolve the issue of the boundedness of the potential. On the other hand, the logarithmic divergences must be understood to fully address the question of soft SUSY breaking. Neglecting radiative corrections to condensate effects (i.e., to $\bar{\lambda}\lambda$ couplings for $\mu > \Lambda_c$), one finds[49] contributions from nonrenormalizable interactions to gaugino masses that are of order $(4\pi)^{-2}m_{\tilde{G}}^3$ and $(4\pi)^{-2}\Lambda^2 m_{\tilde{G}}$. A complete evaluation of the quadratically divergent contributions would at least determine whether or not terms of the second type are present and set a bound on one-loop gaugino masses.

## 2.4 Possibilities for a viable phenomenology.

Let me first summarize the results[47,49] of the preceding sections.

In the model studied above it was found that if the one-loop effective potential is not positive semi-definite everywhere it is unbounded from below, resulting in an infinite, negative cosmological constant and infinite gravitino mass - clearly a physically unacceptable solution. If the potential is bounded, the ground state vacuum energy vanishes. One possibility is that the ground state is uniquely determined with $m_{\tilde{G}} = 0$ and unbroken supersymmetry. This is equally unacceptable since we live in a vacuum that is noninvariant under SUSY. A numerical analysis[47,49] of the potential shows that there are plausible values of the parameters for which an acceptable vacuum, with broken SUSY, a finite gravitino mass and no cosmological constant, can occur. In this case the vacuum has an infinite degeneracy, and the scales $m_{\tilde{G}}, \Lambda_c$ and $\Lambda_{GUT}$ remain undetermined, although their ratios are fixed. The degeneracy is lifted by fixing, for example, the parameter $c$ that appears in the effective potential, Eq. (2.9a). If this parameter is interpreted† as proportional to the vev of the 10-d three-form, Eq. (2.4a), then all scales are determined by the topology of the compact manifold. Furthermore, the quantization condition (2.4b) suggests that the vacuum energy is discrete, and therefore does not have an associated, massless Goldstone mode.

Assuming the existence of a vacuum with finite $m_{\tilde{G}}$, the effective one-loop Lagrangian can be expanded to determine whether effective soft SUSY breaking terms are generated in the observable sector. No such terms are found to be generated by one-loop corrections in the effective theory for $\mu < \Lambda_c$. However, the potential can be bounded and positive semi-definite only if we include loop corrections from physics at scales $\Lambda_c < \mu < \Lambda_{GUT}$, and a complete evaluation of their effects requires further study. The heuristic arguments of Sect. 2.3 suggest that no soft SUSY breaking terms are generated in the effective one-loop scalar potential.

If, in addition, no gaugino masses are generated, it is difficult to guess the origin, or estimate the magnitude relative to $m_{\tilde{G}}$, of SUSY breaking effects in the observable sector, in particular the ratio $m_\varphi/m_{\tilde{G}}$ that governs the gauge hierarchy discussed in Sect. 1.5. It could be that scalar masses arise only in a very high loop order and are therefore suppressed by many powers of the effective loop expansion parameter $1/16\pi^2$.

Alternatively they might be dominated by effects of higher string and/or Kaluza-Klein modes and thus suppressed by powers of $m_{\tilde{G}}/m_P$ and/or $\alpha' m_{\tilde{G}}^2$, where $\alpha'$ is the inverse string tension: $\alpha' \lesssim m_P^{-2}$. In either case the observed gauge hierarchy might be realized but certainly cannot be calculated with present technology.

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† An alternative interpretation, in terms of the vev of a scalar field, has recently been proposed.[64]

If, instead, quantum corrections from scales $\Lambda_c < \mu < \Lambda_{GUT}$ generate nonvanishing gaugino masses at one loop, they are either of order

$$m_{\tilde{g}} \sim m_{\tilde{G}}^3/(4\pi)^2 m_P^2 \tag{2.64a}$$

or of order

$$m_{\tilde{g}} \sim \Lambda^2 m_{\tilde{G}}/(4\pi)^2 m_P^2 \lesssim 100 m_{\tilde{G}}/(4\pi)^2 m_P^2 \tag{2.64b}$$

where I have used the result

$$m_{\tilde{G}} \sim 0.3\Lambda_c \sim 0.1\Lambda_{GUT} \sim (10^{-4} - 10^{-2})/\sqrt{cb_0}. \tag{2.65}$$

As explained in Sect. 1.5, (2.64a) requires

$$m_{\tilde{G}} \lesssim 10^{-4} m_P \tag{2.66a}$$

for a viable gauge hierarchy, while (2.64b) requires

$$m_{\tilde{G}} \lesssim 10^{-6} m_P. \tag{2.66b}$$

If the parameter $c$ is proportional to the vev of $H_{lmn}$, Eq.(2.4a), the quantization condition (2.4b) implies a quantization condition for $c$ of the form[40]

$$\frac{c}{16\pi\sqrt{3\pi}} \left(\frac{m_P}{2\pi^{1/2}}\right)^3 \int_S d\Sigma^{lmn} \epsilon_{lmn} = 2\pi n \tag{2.67}$$

where $\epsilon_{lmn}$ is the anti-symmetric Levi-Civita tensor and I use complex coordinates for the compact 6-manifold: $\epsilon_{\bar{l}\bar{m}\bar{n}} = (\epsilon_{lmn})^*$. In writing (2.4b) and (2.67) the metric of the compact manifold $\mathcal{M}$ has been normalized by defining[40]

$$g_{lm} = e^{\sigma} g_{lm}(0)$$

$$\int_{\mathcal{M}} d^3x d^3\bar{x} \det g_{lm}(0) = \left(\frac{m_P}{2\pi^{1/2}}\right)^6. \tag{2.68}$$

Then one expects

$$I \equiv \left(\frac{m_P}{2\pi^{1/2}}\right)^3 \int_S d\Sigma^{lmn} \epsilon_{lmn} \lesssim 1 \tag{2.69}$$

which implies for $n \neq 0$:

$$c = 10^3 n/I \gtrsim 1000. \tag{2.70}$$

Using the range of values (2.57) for $b_0$ gives

$$m_{\tilde{G}} \simeq \sqrt{\frac{I}{n}}(0.4 - 12) \times 10^{-3} \tag{2.71}$$

which may, from (2.70) be consistent with the requirements (2.66) for a viable gauge hierarchy. It is also interesting that a value as large as (2.70) for $c$ might also allow for a successful inflationary scenario.[87]

The model studied in the preceding sections is in fact a toy model when interpreted as emerging from the compactification of ten-dimensional supergravity. The topology of the compact manifold is characterized by Hodge numbers $h_{ij}$ that are positive integers and determine[45] the spectrum of massless states (before SUSY breaking).

In particular the number of matter generations is given by $b_{11} - b_{21}$; observation therefore requires $b_{11} \geq 3$. In addition to the scalar field $S$, there are a total of $b_{11}$ gauge nonsinglets $T_i$, whereas only one ($T$) was included in the above model. One should therefore pin point the qualitative features of the model studied that assure desirable features at one loop and try to identify a class of more realistic models that incorporate the same features.

As I will explain more explicitly below, the sufficient ingredients[42] to ensure vanishing gauge nonsinglet masses at one loop are a) a partial invariance of the effective tree Lagrangian under a noncompact Heisenberg group $G_H$ of nonlinear transformations, b) a "no-scale" structure[39] of the tree potential, and c) vanishing vacuum energy at tree level. In this context I define "no-scale" by the absence of a term in the potential proportional to $e^{\mathcal{G}}$, which, in the absence of nonperturbative effects, would force an unbroken supersymmetric solution $m_{\tilde{G}} = 0$. In the general class of models that I consider the tree-level vacuum configuration has $\varphi_i = 0$, and its vacuum energy is determined by the contribution (2.9a), defined more generally by

$$U = e^{\mathcal{G}} \left( \frac{\partial^2 \mathcal{G}}{\partial S \partial \bar{S}} \right)^{-1} \left| \frac{\partial \mathcal{G}}{\partial S} \right|^2 . \tag{2.72}$$

Thus the condition for vanishing vacuum energy at tree level is

$$\mathcal{G}_S \equiv \frac{\partial \mathcal{G}}{\partial S} = 0. \tag{2.73}$$

The vanishing of the cosmological constant at one-loop for the model studied above follows essentially from dimensional analysis and therefore should be a feature of a much more general class of models. Finally, the vanishing of A-terms - and possibly gaugino masses - at one-loop, is intimately connected with the vanishing of the cosmological constant. There is no reason why this result should not generalize to more realistic models that incorporate the features a), b), and c) enumerated above, although at present we have no understanding of it in terms of symmetries.

To see how these conditions assure the vanishing of gauge nonsinglet scalar masses at one-loop, recall first (Sect. 1.5) that exact invariance under $G_H$ implies $m_\varphi = 0$ to all orders. This invariance is broken by both the superpotential $W(\varphi)$ and the gauge interactions. In a broken SUSY theory, the latter will induce scalar masses, via the diagrams of Fig. 11(b), of order

$$m_S^2 \sim \frac{\alpha}{4\pi} m_{\lambda}^2. \tag{2.74}$$

In most superstring-inspired models, as in the toy model studied above, the tree-level gaugino masses are determined by the S-field:

$$m_\lambda = e^{\mathcal{G}} (S + \bar{S}) \frac{\partial \mathcal{G}}{\partial S} \tag{2.75}$$

and vanish when the condition (2.73) for a vanishing cosmological constant at tree-level is satisfied. The presence of a superpotential $W(\varphi)$ induces the contributions shown in Fig. (11a) to the scalar self energy. By supersymmetry they cancel identically for vanishing scalar and chiral fermion tree-level masses.

In order to generate nonvanishing gauge nonsinglet scalar masses one needs the interplay of a $G_H$ breaking interaction ($W(\varphi) \neq 0$) with a SUSY breaking interaction (e.g., $W(S) \neq 0$). An analysis[43] of the possible contributions to scalar masses shows that they vanish if Eq. (2.73) is satisfied. This is a one-loop argument only. The conventional wisdom is that gauginos acquire masses at one loop and therefore that scalars will acquire masses, Eq. (2.74), at the two-loop level. If, however, one-loop contributions to gaugino masses vanish, as suggested by the study of contributions from scales $\mu < \Lambda_c$ where the theory is unambiguously specified, it is unclear whether scalars will acquire masses at higher loops. A more thorough understanding, in terms of symmetries, is needed to better address this question.

Since, on the other hand, the vanishing of scalar masses can be understood in terms of a partial Heisenberg symmetry $G_H$, we can ask whether any potentially realistic models possess this partial symmetry. It has been shown[43] that $G_H$ is a remnant of a partial symmetry, which is exact for vanishing gauge couplings, of ten-dimensional supergravity. Under this symmetry the gauge fields $A_M$ and the antisymmetric field $B_{MN}$ (of which the three-form $H_{LMN}$, Eq. (2.4), is the covariant derivative) transform according to:

$$A_M^a \to A_M^a + H_M^a,$$
$$B_{MN} \to B_{MN} + \frac{1}{\sqrt{2}} A^a_{[M} H^a_{N]}, \tag{2.76}$$

where $H_M$ is a harmonic form. In Calabi-Yau compactification,[44] where the $SU(3)$ subgroup of one $E_8$ is identified with the holonomy group of the compact manifold, the limit of vanishing gauge coupling constant is singular, and the appropriate invariance under $G_H$ may not survive[43] in the effective 4-d theory. However, it is expected to survive for orbifold compactification.

Quite generally, consider an effective 4-d Kähler potential of the form

$$\mathcal{G} = G(T,\bar{T},C,\bar{C}) + G_{(S)}(S,\bar{S}) + \ln|W(C) + W(S)|^2 \tag{2.77}$$

where $W(S) \neq 0$ induces tree-level SUSY breaking, and

$$G(T,\bar{T},C,\bar{C}) = -\sum_{A=1}^{n_A} Q_A \ln U_A - \sum_{B=1}^{n_B} P_B \ln\det U^B. \tag{2.78a}$$

The functions $U_A$ are of the form

$$U_A = T_A + \bar{T}_A - \sum_i \bar{C}^{iA} C_{iA} \tag{2.78b}$$

and the $L_B \times L_B$ matrices $U^B$ are of the form

$$U^B_{IJ} = T^B_{IJ} + \bar{T}^B_{IJ} - \sum_a C^{aB}_I \bar{C}^{aB}_J. \tag{2.78c}$$

In Eqs. (2.77), (2.78) the fields $S, T_A$ and some of the $C$'s are gauge singlets. The superpotential defined in this way yields an effective tree-level potential of the form:

$$V = e^{\mathcal{G}} \mathcal{G}_S^S |\mathcal{G}_S|^2 + e^{\mathcal{G}} n + \mathcal{D} + \hat{V}, \tag{2.79}$$

where

$$n = \sum_A Q_A + \sum_B P_B L_B - 3 \tag{2.80}$$

and $D$ are $\hat{V}$ are, respectively, the usual D- and F- terms that are quartic in the gauge nonsinglet fields. The criteria enumerated above, that assure vanishing one-loop scalar masses, are satisfied for $\mathcal{G}_S = n = 0$. Specific examples, based on orbifold compactification, of theories satisfying these criteria have been given by Ferrara et al.[41] with field content and Kähler potential specified by the following table:

| $n_A$ | $n_B$ | $Q$ | $P$ | $L$ |
|---|---|---|---|---|
| 0 | 1 | - | 1 | 3 |
| 3 | 0 | 1,1,1, | - | - |
| 2 | 0 | 1,2 | - | - |
| 1 | 1 | 1 | 1 | 2 |

The existence of these effective theories suggest that a superstring theory in ten dimensions might yield an effective field theory in four dimensions with a realistic particle spectrum and the possibility of generating the hierarchy of scales needed to understand the observed scale of electroweak symmetry breaking.

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