1. p-branes (intermediate)

The mechanics of a generic p-brane (in the sense of an object with $p+1$ dimensional worldvolume) can be described by the Dirac-type action

$$S = -T \int d^{p+1}\xi \sqrt{-\det \gamma},$$

where we have $p+1$ coordinates ξ^α with $\alpha = 0, \ldots, p$. The matrix $\gamma_{\alpha\beta}$ is the *pullback* of the Minkowski metric onto the brane

$$\gamma_{\alpha\beta} = \frac{\partial X^\mu}{\partial \xi^\alpha} \frac{\partial X^\nu}{\partial \xi^\beta} \eta_{\mu\nu}.$$

Show that this action is equivalent to the Polyakov-style action

$$S = -\frac{T}{2} \int d^{p+1}\xi \sqrt{-\det g}\big[g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu - (p-1)\big],$$

where $g_{\alpha\beta}$ is the dynamical worldvolume metric. Do these actions look familiar to you in the cases $p = 0, 1$?

2. Stretched strings (intermediate)

An open string can stretch between two Dp-branes. In fact there are four possibilities for a string to stretch between two branes, called *sectors*. Two of the sectors are strings beginning and ending on the same branes denoted [11] and [22]. The sectors [12] and [21] contain the cases where the string streched between the brane. The last two cases are different because orientation matters. We will be interested in the case where the endpoints of the string lie on two different branes.

a) Write down the mode expansion for a string stretched between two parallel Dp-branes and interpret the result.

b) How does the distance between the branes affect the spectrum of the string? What happens for coincident branes? *Hint:* Consider the mass-squared.

⟶

3. Orbifolds (intermediate – hard)

After having seen how a string behaves under compactification of one dimension, we want to find out how it behaves under restricting it to the half line $x^{D-1} \geq 0$ by the identification

$$x^{D-1} \sim -x^{D-1}.$$

Such a space is called *orbifold* in string theory. Again we abbreviate the relevant coordinate $X(\xi) := X^{D-1}(\xi)$, and introduce an operator U acting as ($\mu = 0, \dots D-2$)

$$UX(\sigma)U^{-1} = -X(\sigma+\pi), \qquad UX^{\mu}(\sigma)U^{-1} = X^{\mu}(\sigma+\pi).$$

U is a symmetry of the orbifold theory, so only states invariant under U are physical.

a) How does U act on the modes x, p, α_n^{L} and α_n^{R} of the coordinate X?

b) Define the string vacuum $|0; q^{\mu}, r\rangle$ where $\mu = 0, \dots, D-2$ and r is the momentum in the folded dimension. We assume that

$$U|0; q^{\mu}, 0\rangle = |0; q^{\mu}, 0\rangle.$$

Give the action of U on $|0; q^{a}, r\rangle$ and write down the ground states of the orbifold theory.

c) What are the massless states of the orbifold theory?