Statistical Physics
Exercise 1

Exercise 1.1 Thermodynamics of a magnetic system

a) Consider a long, empty coil of length L, cross-section A, and N turns with a current I flowing. We now fill the coil uniformly with a paramagnetic material. Show that the work (going into the paramagnet) in the infinitesimal time interval dt is

$$\delta W_{\mathrm{m}} = \boldsymbol{H} \cdot d\boldsymbol{\mathcal{M}} \;, \tag{1}$$

where $\boldsymbol{\mathcal{M}} = \Omega \boldsymbol{M}$ denotes the magnetization, $\boldsymbol{M}$ the magnetization density, and Ω the volume of the paramagnetic material. *Hint:* Use Ampere's and Faraday's law.

b) Consider a rigid, permanent magnetic dipole with uniform magnetization density $\boldsymbol{M}$ (throughout the volume Ω) in an external magnetic field $\boldsymbol{H}$. Show that for an infinitesimal displacement $\mathrm{d}\boldsymbol{l}$ of the dipole the work (going into the system) locally can be written as

$$\delta W_{\mathrm{d}} = -\boldsymbol{M} \cdot d\boldsymbol{H} = -\boldsymbol{M} \cdot d\boldsymbol{B} \;. \tag{2}$$

c) Consider a magnetic system as described in a) or b). Show that the following Maxwell relations hold:

$$\left(\frac{\partial T}{\partial \mathcal{M}}\right)_S = \left(\frac{\partial H}{\partial S}\right)_{\mathcal{M}} \;, \tag{3}$$

and

$$\left(\frac{\partial \mathcal{M}}{\partial T}\right)_H = \left(\frac{\partial S}{\partial H}\right)_T \;. \tag{4}$$

Hint: Identify the magnetic system with a 'simple fluid' (i.e., $H \hat{=} -p$, $\mathcal{M} \hat{=} V$) and use the Maxwell relation of the corresponding potentials.

Exercise 1.2 Ideal paramagnet

In this exercise we study the thermodynamics of an ideal classical paramagnet of unit volume specified by the thermal and the caloric equation of state:

$$M(T,H) = Nm\left[\coth\left(\frac{mH}{k_BT}\right) - \frac{k_BT}{mH}\right] \;, \tag{5}$$

$$U(T,H) = C_M T \;, \tag{6}$$

where m denotes the magnetic moment. From part *a*) of the previous exercise we know that $dU = \delta Q + HdM$.

a) Find the curves of the reversible adiabatics and isotherms in the M-H and in the M-T diagram for the cases (i) $mH \gg k_BT$ and (ii) $mH \ll k_BT$.

Hint: Use $\coth(x) \approx 1/x + x/3$ for $x \ll 1$ and $\coth(x) \approx 1$ for $x \gg 1$.

b) Construct a Carnot engine using the ideal paramagnet as an operating material between two reservoirs 1 and 2 of temperature T_1 and T_2, respectively ($T_1 > T_2$). Calculate the efficiency of the engine for the two cases (i) and (ii) in a).

c) Calculate the entropy $S(U,M)$ for the two cases (i) and (ii) in a).

Office hour: Monday, September 24, 8-10 am (Sarah Etter, HIT K 12.2)