Statistical Physics
Exercise Sheet 13

Exercise 13.1 Magnetic domain wall

We want to calculate the energy of a magnetic domain wall in the framework of the Ginzburg-Landau (GL) theory. Assuming translational symmetry in the (y, z)-plane, the GL functional in zero field reads

$$F[m, m'] = F_0 + \int dx \left\{ \frac{A}{2} m(x)^2 + \frac{B}{4} m(x)^4 + \frac{\kappa}{2} [m'(x)]^2 \right\}. \tag{1}$$

a) Solve the GL equation with boundary conditions

$$m(x \to \pm\infty) = \pm m_0, \quad m'(x \to \pm\infty) = 0, \tag{2}$$

where m_0 is the magnetization of the uniform solution.

b) First, find the energy of the uniformly polarized solution (no domain walls). Next, compute the energy of the solution with a domain wall compared to the uniform solution. Use the coefficients A, B and κ according to the expansion of the mean-field free energy of the Ising model (see Eqs. (5.78) and (5.83)). Finally, find the energy of a sharp step in the magnetization and compare it to the above results.

Exercise 13.2 Linear response of the quantum harmonic oscillator

We consider a quantum harmonic oscillator with charge $q \equiv 1$ subject to an external electric field $e(t)$ assumed small enough to be treated in linear response (dipole approximation). The resulting Hamiltonianis given by

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}' = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2}{2}\hat{x}^2 - e(t)\hat{x}. \tag{3}$$

The response function is defined through

$$\langle \hat{x} \rangle (t) = \int \mathrm{d}t' \chi(t - t') e(t'). \tag{4}$$

a) Find the susceptibility $\chi(\omega)$ and the dynamical structure factor $S(\omega)$.
Interpret the spectrum of $S(\omega)$.

Hint: Use the Kubo formalism (see Ch. 6.1)

$$\chi(\omega) = \sum_{n,n'} \frac{e^{-\beta\epsilon_n}}{Z} |\langle n|\hat{x}|n'\rangle|^2 \left(\frac{1}{\hbar\omega - \epsilon_{n'} + \epsilon_n + i\hbar\eta} - \frac{1}{\hbar\omega + \epsilon_{n'} - \epsilon_n + i\hbar\eta} \right), \tag{5}$$

$$S(\omega) = \sum_{n,n'} \frac{e^{-\beta\epsilon_n}}{Z} |\langle n|\hat{x}|n'\rangle|^2 \delta(\hbar\omega - \epsilon_{n'} + \epsilon_n). \tag{6}$$

*b) For an external electric field of the form $e(t) = \tilde{e}(\omega)\cos\omega t$, switched on at $t = t'$, find the dissipated power $\overline{\mathrm{d}E/\mathrm{d}t}(\omega)$ and the time dependence of the susceptibility $\chi(t - t')$. How does causality come into play?

Office Hour: Monday, December 17th, 8:00-10:00 K12.2 (Sarah Etter)