Sheet IV

`Return by 17.10.2013`

Question 1 [*Representation theory of S_4 and characters - part II*]: Using the known results for the characters of S_3 and S_4, decompose the irreducible representations of S_4 in terms of the irreducible representations of S_3.

Question 2 [*Branching rules from O to D_4*]: In the lecture the branching rules from O to D_3, i.e. the decomposition of irreducible O-representations into D_3-representations, were computed. In particular, they allowed one to describe qualitatively the splitting of the energy eigenstates with angular momentum number $L = 0, 1, 2, 3$ under a crystal-field perturbation that preserves the symmetry around a diagonal.

In this exercise you should perform the corresponding analysis for the case when the crystal-field perturbation breaks the octahedral symmetry down to the symmetries that preserve one of the four-fold axes of the group O.

[*Hint:* The residual symmetry group is then D_4. Be careful to identify the conjugacy classes of D_4 inside those of O.]

Question 3 [*Vanishing of characters*]: A general theorem in character theory states that the character of any irreducible representation R of dimension greater than 1 assumes the value 0 on some conjugacy class of the group, i.e. there exists some conjugacy class C such that $\chi_R(g) = 0$ for all $g \in C$.

Prove the above statement with the additional assumption that the character of the corresponding representation takes values in $\mathbb{Z}$, i.e. that $\chi_R(g) \in \mathbb{Z}$ for all $g \in G$.