Sheet IX

Return by 28.11.2013

Question 1 [*Clebsch-Gordon series for* $\mathfrak{su}(2)$]: Derive the Clebsch-Gordon series for $\mathfrak{su}(2)$

$$V^{(n)} \otimes V^{(m)} = \bigoplus_{l=|n-m|}^{n+m} V^{(l)} \ .$$

Question 2 [*Cartan-Weyl basis of* $\mathfrak{su}(3)$]: Write out explicitly the commutation relations of $\mathfrak{su}(3)$ in the Cartan-Weyl basis.

Question 3 [*Representation of* $\mathfrak{su}(3)$]: Construct explicitly the representation R of $\mathfrak{su}(3)$ that is generated from the highest weight state v, i.e., the state satisfying

$$E_{12}(v) = E_{13}(v) = E_{23}(v) = 0 \ ,$$

with the eigenvalues

$$H_{12}(v) = 2v \ , \qquad H_{23}(v) = 0 \ .$$

Determine, in particular, the dimension of R and the eigenvalues (with multiplicities) of H_{12} and H_{23}. Proceed as follows:

(i) Show that R is spanned by the vectors $w(v)$, where w is any word in E_{21} and E_{32}.

(ii) Acting on v with E_{21} and E_{32}, construct a basis of R. For any new vector, compute its eigenvalues under H_{12} and H_{23}, and verify that you can go back to the vectors previously constructed (and thus, by recursion, to v), by acting with some element of $\mathfrak{su}(3)$. If this is not possible the vector must be 0, since by assumption the representation is irreducible.