Exercise 1. $2 \to 2$ ***scattering***

The cross section for the process $a(p_a, m_a)\, b(p_b, m_b) \to c(p_c, m_c)\, d(p_d, m_d)$ can be written in terms of the squared matrix element $|M|^2$ as

$$\sigma = \frac{1}{f} \int \frac{d^3\vec{p}_c}{(2\pi)^3 2E_c} \frac{d^3\vec{p}_d}{(2\pi)^3 2E_d} (2\pi)^4 \delta(p_a + p_b - p_c - p_d)\, |M|^2$$

where $f = 4[(p_a \cdot p_b)^2 - m_a^2 m_b^2]^{1/2}$ is the flux factor.

Show that the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{\lambda^{1/2}(s, m_c^2, m_d^2)}{\lambda^{1/2}(s, m_a^2, m_b^2)}\, |M|^2$$

where $\lambda(x_1, x_2, x_3) \equiv x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_2x_3$.

Exercise 2. $e^+ e^- \to \mu^+ \mu^-$

In the lecture the differential cross section for the process $e^- \mu^- \to e^- \mu^-$ at order $\mathcal{O}(e_0^4)$ was computed in the high energy limit as

$$\frac{d\sigma}{d\Omega} = \frac{e_0^4}{32\pi^2 s} \frac{s^2 + u^2}{t^2}$$

Use this result and crossing symmetry to show that the total cross section for $e^+ e^- \to \mu^+ \mu^-$ in the high energy limit is given by

$$\sigma(e^+ e^- \to \mu^+ \mu^-) = \frac{4\pi\alpha^2}{3\, s}$$

where the fine-structure constant is defined as $\alpha \equiv e^2/(4\pi)$.

Exercise 3. ***Bhabha scattering*** $e^+ e^- \to e^+ e^-$

Draw all Feynman diagrams for the process $e^+ e^- \to e^+ e^-$ and the corresponding cut diagrams for the amplitude squared.

Evaluate the cut diagrams and show that the spin summed/averaged matrix element squared for the process $e^+ e^- \to e^+ e^-$ in the high-energy limit is given by

$$\langle |M|^2 \rangle = 2e_0^4 \left(\frac{t^2 + u^2}{s^2} + \frac{s^2 + u^2}{t^2} + \frac{2\, u^2}{s\, t} \right)$$

What is the total cross section for this process in the high-energy limit?

Hint: recycle previous calculations/results as much as possible.