Phase Transitions and Critical Phenomena

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Exercise Sheet 4

HS 14
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Problem 1 Correlation function $G(\boldsymbol{r})$ in $d = 3$

The correlation function of order parameter $\eta(\boldsymbol{r})$ is defined as

$$G(\boldsymbol{r} - \boldsymbol{r}') = \langle \eta(\boldsymbol{r})\eta(\boldsymbol{r}') \rangle - \langle \eta(\boldsymbol{r}) \rangle \langle \eta(\boldsymbol{r}') \rangle \tag{1}$$

where $\langle \ldots \rangle$ denotes the statistical average. In the lecture we derived that the Fourier transform of the correlation function is given by

$$G(\boldsymbol{k}) = \frac{T}{\gamma \left(\boldsymbol{k}^2 + \xi^{-2} \right)} \tag{2}$$

where $\xi \propto |T - T_{\mathrm{c}}|^{-\nu}$ is the correlation length. Use result (2) to derive the real space correlation function $G(\boldsymbol{r})$ in $d = 3$.

Problem 2 Superconducting grain

Consider a superconducting grain of size L that is much smaller than the correlation length ξ. Under such circumstances we can drop the gradient term in the free energy and the partition function of the system is

$$Z = \int d\psi d\psi^* \exp \left[-\beta \left(at \, |\psi|^2 + b \, |\psi|^4 \right) \right]. \tag{3}$$

Use this expression to find the specific heat of the grain around the phase transition.