Introduction to Superconductivity
Exercise Sheet 7

Discussion date: 12 November 2014

Exercise 1: Order parameter of an isolated vortex.

In the lecture you have derived an approximation for the magnetic field of a vortex. In this exercise sheet you will calculate the form of its order parameter.

We want to find the structure of an isolated vortex with n quanta in the two limits of (i) the core region $r \ll \xi$ and (ii) far away from the core $\xi \ll r \ll \lambda$. It is most convenient to use cylindrical coordinates, with the z direction along the vortex, and the origin of the polar plane at the center of the vortex. The system is translationally invariant along the z-direction, so we only have to consider the plane (r, θ).

(a) Derive the first Ginzburg Landau equation using the following ansatz for the vortex wave function

$$\Psi = \Psi_\infty f(r) e^{in\theta}, \tag{1}$$

where $|\Psi_\infty|^2 = -\alpha/\beta$ and $f(r) = |\Psi(r)|/|\Psi_\infty|$.

(b) Write the vector potential in terms of the magnetic field $b(r)$, assuming that $A(r,\theta) = A(r)\hat{\theta}$. The gauge is already fixed by the choice of the phase of the order parameter.

(c) What is $A(r)$ in the two limits of the core region $r \ll \lambda$ and very far away $r \gg \lambda$?

(d) What is the structure of the vortex $f(r)$ in the core region $r \ll \xi$?

Comments:

For the lowest order we assume $f(r) = cr^\alpha$ (Taylor) and we can neglect the vector potential (why?). Consider only the most singular terms to determine how the power α depends on the vorticity n. Is it possible to determine the prefactor c?

For the next order we make an extended ansatz $f(r) = cr^\alpha + \delta r^\beta$, with $\beta > \alpha$. Now the vector potential cannot be neglected, but you can use your result from (c).

(e) What is the structure of the vortex $f(r)$ far away from the core $\xi \ll r \ll \lambda$?

Comment:

Very far away from the vortex $r \gg \lambda$ we have $f \to 1$. For the intermediate region $\xi \ll r \ll \lambda$ we make an ansatz $f = 1 - \delta f$. We can again neglect the vector potential (why?). Solve the equation for δf.

(f) What is the healing behavior of the order parameter?

(g) There are approximate solutions trying to capture the structure of $f(r)$ over the whole range

$$f_1(r) \approx \tanh\left(\frac{\nu r}{\xi}\right) \tag{2}$$

$$f_2(r) \approx \frac{r}{\sqrt{r^2 + 2\xi^2}} \tag{3}$$

where $\nu \approx 1$. Plot these approximations and your results, and compare.