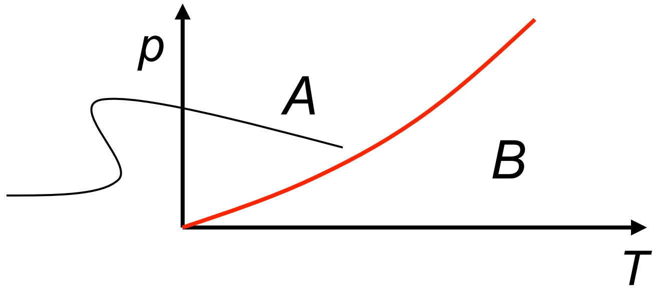
Ehrenfest relation for 2nd order phase transition

transition between phase *A* and *B*

$$\left.\begin{aligned} S_A(T,p) &= S_B(T,p) \\ V_A(T,p) &= V_B(T,p) \end{aligned}\right\} \text{ continuous}$$



differentials

$$\begin{cases} \left(\frac{\partial S_A}{\partial T}\right)_p dT + \left(\frac{\partial S_A}{\partial p}\right)_T dp = \left(\frac{\partial S_B}{\partial T}\right)_p dT + \left(\frac{\partial S_B}{\partial p}\right)_T dp \,, \\ \left(\frac{\partial V_A}{\partial T}\right)_p dT + \left(\frac{\partial V_A}{\partial p}\right)_T dp = \left(\frac{\partial V_B}{\partial T}\right)_p dT + \left(\frac{\partial V_B}{\partial p}\right)_T dp \,. \end{cases}$$

Maxwell relation

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p = -V\alpha$$

thermal expansion coefficient

Ehrenfest relations

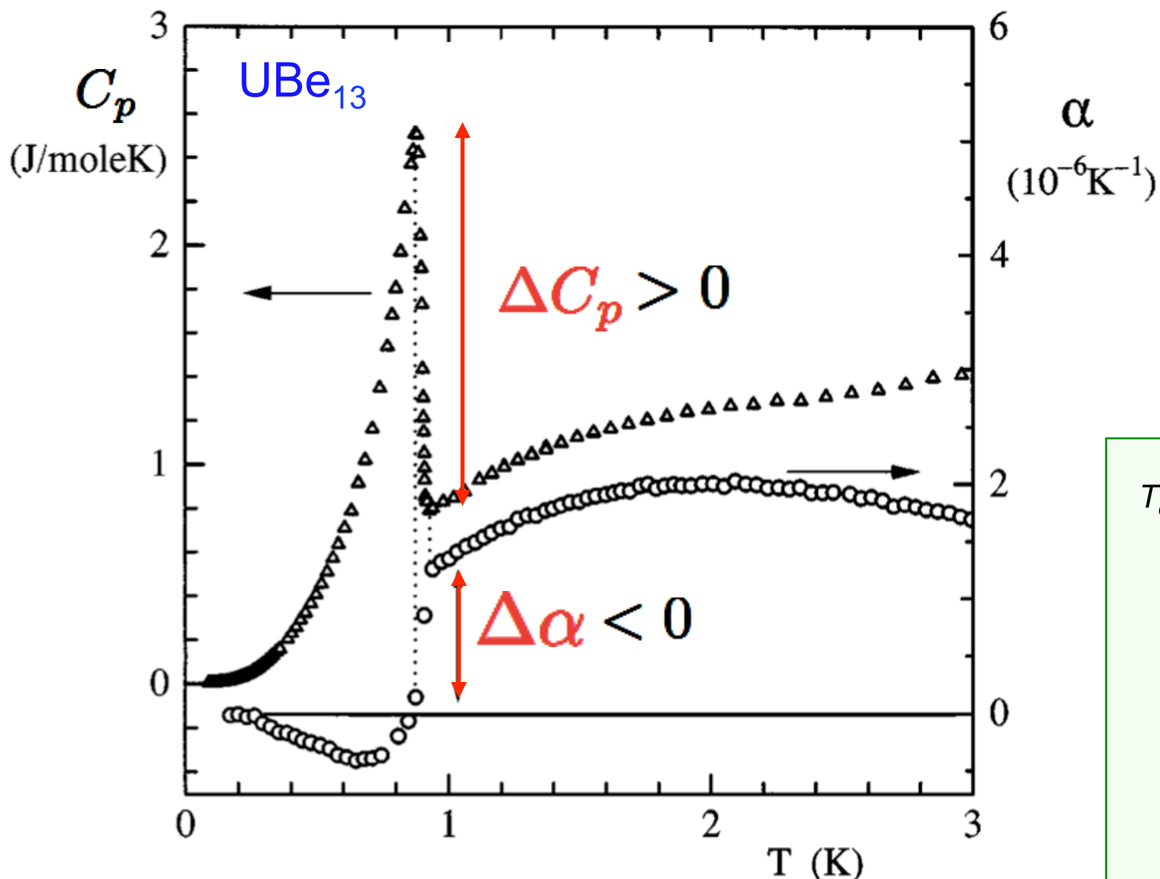
experimentally testable

$$\frac{dp}{dT} = -\frac{\left(\frac{\partial S_B}{\partial T}\right)_p - \left(\frac{\partial S_A}{\partial T}\right)_p}{\left(\frac{\partial S_B}{\partial p}\right)_T - \left(\frac{\partial S_A}{\partial p}\right)_T} = \frac{\Delta C_p}{TV\Delta\alpha}$$

$$\frac{dp}{dT} = -\frac{\left(\frac{\partial V_B}{\partial T}\right)_p - \left(\frac{\partial V_A}{\partial T}\right)_p}{\left(\frac{\partial V_B}{\partial p}\right)_T - \left(\frac{\partial V_A}{\partial p}\right)_T} = \frac{\Delta\alpha}{\Delta\kappa_T}$$

Ehrenfest relation for 2nd order phase transition

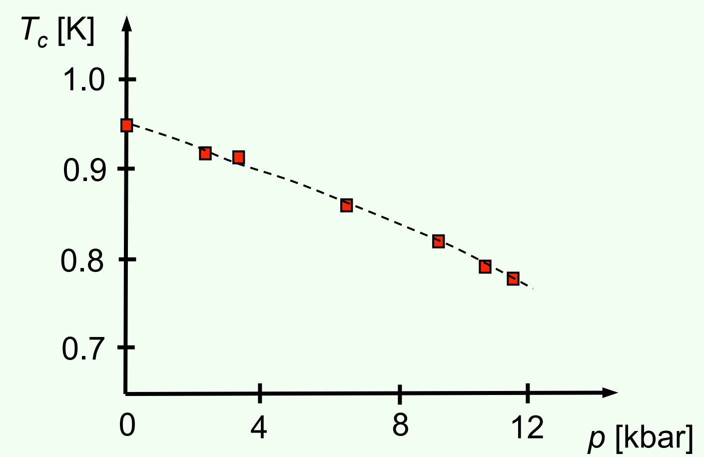
superconductor



F. Kromer et al., Phys. Rev. Lett. 81, 4476 (1998)

pressure dependence of T_c

$$\frac{dT_c}{dp} = \frac{TV\Delta\alpha}{\Delta C_p} < 0$$



S.E. Lambert et al.
Phys. Rev. Lett. 57, 1619 (1986)