$$\mathcal{H} = \hbar\omega \left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right) = \hbar\omega \left(\hat{n} + \frac{1}{2}\right) \qquad \frac{\mathcal{H}|n\rangle = \epsilon_n|n\rangle}{\epsilon_n = \hbar\omega(n+1/2)}$$

raising
$$\hat{a}^\dagger |n
angle = \sqrt{n+1} |n+1
angle$$
 create particle with $E=\hbar \omega$

lowering
$$|\hat{a}|n
angle=\sqrt{n}|n-1
angle$$
 delete particle with $E=\hbar\omega$

quantization:
$$[\hat{a},\hat{a}^\dagger]=1$$
 $[\hat{a},\hat{a}]=[\hat{a}^\dagger,\hat{a}^\dagger]=0$ $\hat{a}^\dagger\hat{a}=\hat{n}$ particle number operator

$$\langle \hat{n} \rangle = rac{1}{Z} \sum_{n=0}^{\infty} n e^{-\beta \epsilon_n} = rac{1}{e^{\beta \hbar \omega} - 1}$$
 "bosonic"

$$\mathcal{H} = rac{1}{2} \left(P^2 + \omega^2 Q^2
ight) \stackrel{ ext{quantization}}{-----} \left[\hat{Q}, \hat{P}
ight] = i \hbar \quad \left\{ egin{array}{l} \hat{Q} = \sqrt{rac{\hbar}{2\omega}} (\hat{a} + \hat{a}^\dagger) \ \hat{P} = i \omega \sqrt{rac{\hbar}{2\omega}} (\hat{a} - \hat{a}^\dagger) \end{array}
ight.$$

$$\mathcal{H} = \hbar\omega \left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right) = \hbar\omega \left(\hat{n} + \frac{1}{2}\right)$$
 $\qquad \frac{\mathcal{H}|n\rangle = \epsilon_n|n\rangle}{\epsilon_n = \hbar\omega(n+1/2)}$

canonical ensemble

$$Z = tre^{-\beta \mathcal{H}} = e^{-\beta \hbar \omega/2} \sum_{n=0}^{\infty} e^{-\beta \hbar \omega n} = \frac{e^{-\beta \hbar \omega/2}}{1 - e^{-\beta \hbar \omega}}$$

internal energy
$$U=-rac{\partial \ln Z}{\partial eta}=rac{1}{2}\hbar\omega+rac{\hbar\omega}{e^{eta\hbar\omega}-1}$$