Critical exponents

singular behavior at $T=T_{c+/-}$

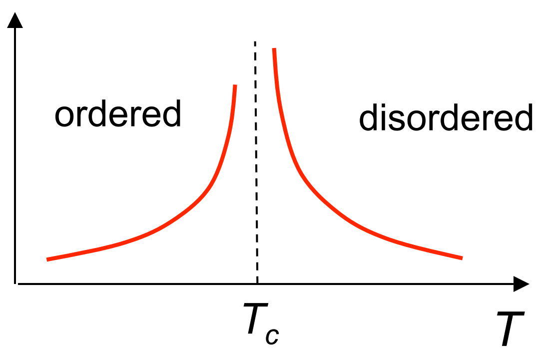
control parameter $\tau = 1 - \frac{T}{T_c}$

$\tau > 0\,, \tau < 0$

heat capacity $C(T) \propto |\tau|^{-\alpha}$

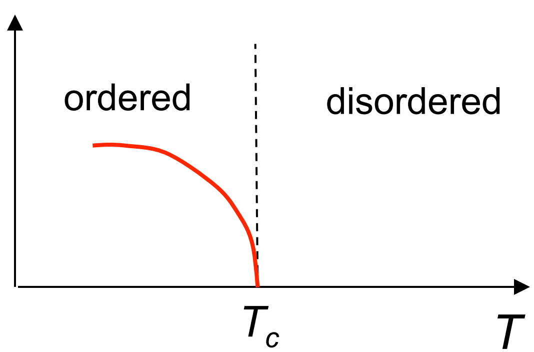
susceptibility $\chi(T) \propto |\tau|^{-\gamma}$

correlation length $\xi(T) \propto |\tau|^{-\nu}$



$\tau > 0 \quad (T < T_c)$

order parameter $m(T) \propto |\tau|^{\beta}$



Critical exponents

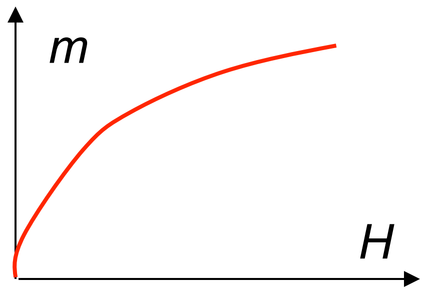
singular behavior at $T=T_{c+/-}$

control parameter $\tau = 1 - \frac{T}{T_c}$

$\tau = 0 \quad (T = T_c)$

order parameter $m \propto H^{1/\delta}$

correlation function $\Gamma_{\vec{r}} \propto \frac{1}{r^{d-2+\eta}}$



scaling laws

Rushbrooke scaling: $\alpha + 2\beta + \gamma = 2$

Widom scaling: $\gamma = \beta(\delta - 1)$

Fisher scaling: $\gamma = (2 - \eta)\nu$

Josephson scaling: $\nu d = 2 - \alpha$

Critical exponents

Fisher scaling: $\gamma = (2 - \eta)\nu$

general correlation function $\Gamma_{\vec{r}} \propto \frac{1}{r^{d-2+\eta}} g(r/\xi)$ $\xi(T) \propto |\tau|^{-\nu}$

susceptibility $\chi(T) \propto |\tau|^{-\gamma}$

$$\chi \propto \int d^d r \, \Gamma_{\vec{r}} \propto \int d^d r \, \frac{1}{r^{d-2+\eta}} g(r/\xi)$$

$$\propto \xi^{2-\eta} \int d^d y \frac{1}{y^{d-2+\eta}} g(y) \propto |\tau|^{-\nu(2-\eta)}$$

Critical exponents

Mean field exponents: $$-A'\tau m + Bm^3 - H - \kappa\vec{\nabla}^2 m = 0$$

$\tau < 0 \quad (T > T_c)$

$$\xi^2 = -\frac{\kappa}{A'\tau} \propto |\tau|^{-2\nu} \qquad \nu = \frac{1}{2}$$

$$\chi = -\frac{1}{A'\tau} \propto |\tau|^{-\gamma} \qquad \gamma = 1$$

$\tau > 0 \quad (T < T_c)$

$$m^2 = \frac{A'\tau}{B} \propto |\tau|^{2\beta} \qquad \beta = \frac{1}{2}$$

$\tau = 0 \quad (T = T_c)$

$$Bm^3 = H \propto H^{3/\delta} \qquad \delta = 3$$

$$\Gamma_{\vec{r}} \propto \frac{1}{r^{d-2}} \propto \frac{1}{r^{d-2+\eta}} \qquad \eta = 0$$

$$C \propto \Theta(\tau) \propto |\tau|^{-\alpha} \qquad \alpha = 0$$

Spontaneous symmetry breaking - long range order

Ginzburg-Landau theory (Ising model of ferromagnet)

order parameter $m \propto \begin{cases} 0 & T > T_c \\ \pm|\tau|^{1/2} & T < T_c \end{cases}$

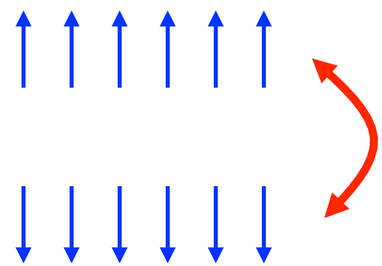
high symmetry $\mathcal{G}$

$\cup$

low symmetry $\mathcal{G}'$

choice between 2 states

two-fold degeneracy



time reversal

broken symmetry

free energy functional

$$F[m; H, T] = \int d^d r \left\{ \frac{A}{2} m(\vec{r})^2 + \frac{B}{4} m(\vec{r})^4 - H(\vec{r}) m(\vec{r}) + \frac{\kappa}{2} [\vec{\nabla} m(\vec{r})]^2 \right\}$$

scalar (invariant) under symmetry operations in $\mathcal{G} = G \times \mathcal{K}$

(G: space group; $\mathcal{K}$: time reversal)

correlation function

$$\Gamma_{ij} = \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \xrightarrow{\vec{r} = \vec{r}_j - \vec{r}_i} \Gamma_{\vec{r}} \propto \frac{e^{-r/\xi}}{r^b}$$

for $T < T_c$, $T > T_c$

$$\lim_{r \to \infty} \Gamma_{\vec{r}} = 0 \Rightarrow \lim_{r \to \infty} \langle s_i s_j \rangle = \langle s_i \rangle \langle s_j \rangle$$

$T > T_c$ $\quad \langle s_i \rangle = 0 \quad$ $\displaystyle \lim_{r \to \infty} \langle s_i s_j \rangle = 0$

$T < T_c$ $\quad \langle s_i \rangle = \pm m \quad$ $\displaystyle \lim_{r \to \infty} \langle s_i s_j \rangle = m^2 > 0$

long range order

correlation over arbitrary distance