This last exercise sheet does NOT count for the Testat (for UZH students),
but it covers topics which potentially can appear in the exam.

Exercise 1. *Hamilton-Jacobi equation*

A particle of charge q is constrained to move in a plane under the influence of a central force potential (non-electromagnetic) $V = \frac{1}{2}kr^2$ and a constant magnetic field $\mathbf{B}$ perpendicular to the plane, so that the vector potential can be expressed as

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}. \tag{1}$$

(a) Set up the Hamilton-Jacobi equation for Hamilton's characteristic function in polar coordinates (r, θ).

Hint. Start with the Lagrangian in cartesian coordinates and change to polar after incorporating the magnetic term. Introduce canonical momenta as usual to obtain the Hamiltonian.

(b) Separate the equation and reduce it to quadratures. Discuss the motion if the canonical momentum p_θ is zero at $t = 0$.

Exercise 2. *Poisson brackets in the Kepler problem*

Show that the components of the Laplace-Runge-Lenz vector

$$\mathbf{A} = \frac{1}{\mu}\mathbf{p} \times \mathbf{L} - \frac{\mathbf{r}}{r} \tag{2}$$

satisfy the following relations

$$\{A_i, A_j\} = -\frac{2E}{\mu^2}\varepsilon_{ijk}L_k \tag{3}$$

where E is the energy of the orbit and μ the reduced mass.

Hint. Make use of the identity $\{f, gh\} = g\{f, h\} + \{f, g\}h$ and exploit the Poisson brackets calculated in the lecture.